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ORIGINAL RESEARCH

# Quantitative supervaluationism

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**Abstract**
So far, the method of supervaluations has been mainly employed to define a non-gradable property of sentences, supertruth, in order to provide an analysis of truth. But it is also possible, and arguably at least as plausible, to define a gradable property of sentences along the same lines. This paper presents a supervaluationist semantics that is quantitative rather than qualitative. As will be shown, there are at least two distinct interpretations of the semantics — one alethic, the other epistemic — which can coherently be adopted to address key issues such as vagueness and future contingents.

**Keywords** Supervaluationism · Verity · Credibility · Vagueness · Future contingents

## 1 Overview

Historically, supervaluationism was developed as an analysis of truth in order to tackle some well known philosophical issues. According to a line of thought due to Mehlberg, Lewis, Kamp, Dummett, and Fine, supervaluationism enables us to deal with vagueness, on the assumption that a vague language is a language that can be made precise in more than one way.[1] According to another line of thought initiated by

[1] Mehlberg (1958), D. K. Lewis (1970), Kamp (1975), Dummett (1978), Fine (1975). A more recent discussion is provided in (Varzi, 2007).

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van Fraassen, supervaluationism provides a semantic treatment of empty names and of self-referential sentences such as those involved in the Liar paradox.[2] According to a third line of thought initially suggested by van Fraassen and then developed by Thomason on the basis of Prior's branching time semantics, supervaluationism yields a plausible account of future contingents.[3] All these accounts rest on the same idea: if a sentence is evaluable as true or false for each element of a set of indices associated with a given parameter, then a non-indexed truth value can be assigned to the sentence relative to that parameter by universally quantifying over such indices. The sentence is *supertrue* when it is true relative to all indices, *superfalse* when it is false relative to all indices, and neither supertrue nor superfalse otherwise.[4]

Although the term 'supervaluationism' is traditionally associated with an analysis of truth along the lines just indicated, there is a more general sense in which this term can coherently be used. Supervaluationism can be understood broadly as a method for defining a property of sentences that hinges on a distinction between two semantic levels marked by two kinds of value assignments. At the first level — the *valuation* level — each sentence, relative to a suitable parameter, receives a value for each element of a set of indices associated with that parameter. At the second level — the *supervaluation* level — the sentence gets a non-indexed value relative to the same parameter on the basis of the indexed values it receives at the first level. The value so obtained expresses the property to be defined.

This broad characterization of supervaluationism leaves room for a crucial distinction. The historical version of supervaluationism as an analysis of truth is *qualitative* in the following sense: the property defined at the supervaluation level is non-gradable. For each of the three values that a sentence can take — supertruth, superfalsity, or neither — either the sentence has it or not. Yet this is not the only way to go. Instead of asking whether a certain value holds for all indices at the valuation level, which is a yes/no question, one can ask *what is the proportional weight* of the indices for which it holds, a different question that allows for degrees. We call *quantitative* any version of supervaluationism that defines some gradable property in the second way. This paper investigates quantitative supervaluationism as distinct from qualitative supervaluationism.

The idea of measuring the indices in which a certain value holds is not new. As Lewis and Kamp suggested long ago in connection with a discussion of comparative adjectives, the method of supervaluations can be employed to define degrees of truth.[5] In a way, the theoretical option that we want to explore has always been there. However, we believe that the potential of quantitative supervaluationism has not been fully appreciated so far, perhaps due to the attention elicited by its qualitative counterpart. In particular, at least three points deserve consideration.

[2] van Fraassen (1966), van Fraassen (1968). The method of supervaluations is considered in Kripke (1975), pp. 711–712, and has later been developed by other authors, see for example Stern (2018).

[3] van Fraassen (1966), Prior (1967), Thomason (1970).

[4] Accordingly, 'supervaluationism' is sometimes contrasted with 'subvaluationism', which is taken to involve existential rather than universal quantification.

[5] D. K. Lewis (1983), p. 229, Kamp (1975), pp. 137–145.

First, quantitative supervaluationism is at least as versatile as qualitative supervaluationism, for it can be applied to the same variety of issues. Here we will focus on vagueness and future contingents, two key issues in which the specificity of quantitative supervaluationism emerges clearly. Second, quantitative supervaluationism admits an epistemic interpretation which is distinct from the more traditional alethic interpretation and yields an intuitively adequate and formally rigorous account of epistemic notions such as rational acceptability. Third, quantitative supervaluationism may be regarded as a generalization of qualitative supervaluationism, in that the latter is definable in terms of the former.

The structure of the paper is as follows. Sections 2 and 3 set out the formal semantics and show its key properties. Sections 4 and 5 present the alethic interpretation of the semantics and explain how it applies to vagueness and future contingents. Sections 6 and 7 present the epistemic interpretation of the semantics and explain how it applies to vagueness and future contingents. Section 8 compares quantitative supervaluationism with qualitative supervaluationism. Section 9 provides some concluding remarks on assertibility.

## 2 Basic notions: valuation

Let L be a language whose alphabet is constituted by a set of sentence letters $p, q, r, \ldots$ and the connectives $\neg, \wedge, \vee$. The formation rules of L are as usual: sentence letters are atomic formulas; if $\alpha$ is a formula, so is $\neg\alpha$; if $\alpha$ and $\beta$ are formulas, so are $\alpha \wedge \beta$ and $\alpha \vee \beta$.

The semantics of L is structured in two distinct levels as required by supervaluationism. At the valuation level, the formulas of L are evaluated relative to parameters that we call *points* and indices that we call *extensions*. For any formula $\alpha$ of L and any interpretation of L, $\alpha$ receives a value relative to each pair $\langle x, e\rangle$, where $x$ is a point and $e$ is an extension associated with $x$. At the supervaluation level, the formulas of L are evaluated relative to points on the basis of what holds at the valuation level. The value of a formula $\alpha$ at a point $x$ is determined by the values that $\alpha$ takes at $\langle x, e\rangle$ for each $e$.

Let us start from the basic ingredients of the semantics.

**Definition 1** *A frame $\mathcal{F}$ for* L *is a triple $\langle X, Y, E\rangle$, where*

- $X$ and $Y$ are non-empty sets;
- $E$ is a function from $X$ to $\mathcal{P}(Y)$ such that, for each $x \in X$, $E(x)$ is countable.

The elements of $X$ are points. The elements of $Y$ are objects that can be assigned to points as their extensions. $E$ assigns to each $x \in X$ a countable subset $E(x)$ of $Y$. For each $e \in E(x)$ we say that $e$ is an extension *of* $x$. The countability condition imposed on $E$, which yields a useful technical simplification, is quite reasonable as a constraint on frames. Arguably, the main results that can be obtained by applying the method of supervaluations to the philosophical issues mentioned above do not require that the set of indices be uncountable, for what matters is that the indices capture some fine-grained distinctions that are taken to be relevant.[6]

**Definition 2** *A model of* L *is a triple* $\langle \mathcal{F}, P, V \rangle$*, where* $\mathcal{F}$ *is a frame* $\langle X, Y, E \rangle$ *while* $P$ *and* $V$ *are functions defined as follows:*

- $P$ assigns to each $x \in X$ a countably additive function $P_x$ from $E(x)$ to $[0, 1]$ in such a way that $\sum_{e \in E(x)} P_x(e) = 1$;
- $V$ assigns 1 or 0 to each atomic formula of L for each pair $\langle x, e \rangle$, where $e \in E(x)$.

$P$ associates with each point $x$ a *proximity assignment* $P_x$: the value that $P_x$ assigns to each extension $e$ of $x$ is intended to measure how "close" $e$ is from the point of view of $x$. $V$ is a function that assigns values to pairs of formulas and point-extension pairs. For each formula $\alpha$ and pair $\langle x, e \rangle$, $V(\alpha, \langle x, e \rangle)$ indicates the value that $V$ assigns to $\alpha$ relative to $\langle x, e \rangle$.

The valuation level can now be defined by specifying a function $v$ that assigns values to the formulas of L relative to point-extension pairs, using the notation $v(\alpha)_{x,e}$ to indicate $v(\alpha, \langle x, e \rangle)$, that is, the value that $v$ assigns to $\alpha$ relative to $\langle x, e \rangle$.

**Definition 3**

1 If $\alpha$ is atomic, $v(\alpha)_{x,e} = V(\alpha, \langle x, e \rangle)$;
2 $v(\neg\alpha)_{x,e} = 1$ iff $v(\alpha)_{x,e} = 0$;
3 $v(\alpha \wedge \beta)_{x,e} = 1$ iff $v(\alpha)_{x,e} = 1$ and $v(\beta)_{x,e} = 1$;
4 $v(\alpha \vee \beta)_{x,e} = 1$ iff $v(\alpha)_{x,e} = 1$ or $(\beta)_{x,e} = 1$.

Let $|\alpha|_x$ be the set of extensions of $x$ such that $v(\alpha)_{x,e} = 1$. One fact to be noted about Definition 3 is that it entails what follows:

**Proposition 1** *For every* $\alpha, \beta$*, and* $x$*,*

(a) $|\alpha \wedge \beta|_x = |\alpha|_x \cap |\beta|_x$;
(b) $|\alpha \vee \beta|_x = |\alpha|_x \cup |\beta|_x$.

*Proof.* (a) follows from clause 3 of Definition 3 given that, for every $e \in E(x)$, $e \in |\alpha|_x \cap |\beta|_x$ iff $v(\alpha)_{x,e} = 1$ and $v(\beta)_{x,e} = 1$. (b) follows from clause 4 of Definition 3 given that, for every $e \in E(x)$, $e \in |\alpha|_x \cup |\beta|_x$ iff $v(\alpha)_{x,e} = 1$ or $v(\beta)_{x,e} = 1$. □

Finally, a relation of logical consequence can be defined in accordance with Definition 3 in terms of preservation of the value 1 for all point-extension pairs.

**Definition 4** $\alpha_1, \ldots \alpha_n \vDash_v \beta$ iff for every pair $\langle x, e \rangle$ in every model, if $v(\alpha_1)_{x,e} =, \ldots, = v(\alpha_n)_{x,e} = 1$, then $v(\beta)_{x,e} = 1$.

[6] An alternative way to define frames is the following: instead of having a set $Y$ distinct from $X$, one can generate extensions out of $X$ by defining a function from $X$ to $\mathcal{P}(\mathcal{P}X)$, that is, a function that assigns to each $x \in X$ a set of elements of $X$ itself.

Leaving aside relativity to point-extension pairs, the relation expressed by the symbol $\vDash_v$ is nothing but the classical relation of logical consequence for a propositional language.

## 3 Basic notions: supervaluation

Once we have the first floor of our semantic building, we may proceed with the second floor. In order to get the supervaluation level we need a different function $sv$ which assigns values to formulas relative to points. $sv$ is defined as follows, using the notation $sv(\alpha)_x$ to indicate the value of $\alpha$ relative to $x$.

**Definition 5**

$$sv(\alpha)_x = \sum\nolimits_{e \in |\alpha|_x} P_x(e)$$

The value of $\alpha$ in $x$ is the sum of the proximity values of the extensions of $x$ in which $\alpha$ is true, which can be understood as the *proportional weight* of these extensions. Since $|\alpha|_x \subseteq E(x)$, and $\sum_{e \in E(x)} P_x(e) = 1$, we get that $sv(\alpha)_x \leq 1$. The limiting case in which $sv(\alpha)_x = 1$ arises when $v(\alpha)_{x,e} = 1$ for every $e \in E(x)$ such that $P_x(e) > 0$. The other limiting case is that in which $sv(\alpha)_x = 0$ because $v(\alpha)_{x,e} = 0$ for every $e \in E(x)$ such that $P_x(e) > 0$. Therefore, for any $\alpha$ and $x$, $0 \leq sv(\alpha)_x \leq 1$.

Here are some key properties of the function $sv$. First, $sv$ behaves as expected with respect to $\vDash_v$:

**Proposition 2** *If* $\vDash_v \alpha$*, then* $sv(\alpha)_x = 1$ *for every* $x$.

*Proof.* Assume that $\vDash_v \alpha$. Then, for any $x$, every $e \in E(x)$ is such that $v(\alpha)_{x,e} = 1$. So $|\alpha|_x = E(x)$, which means that $sv(\alpha)_x = 1$. □

**Proposition 3** *If* $\vDash_v \neg\alpha$*, then* $sv(\alpha)_x = 0$ *for every* $x$.

*Proof.* Assume that $\vDash_v \neg\alpha$. Then, for any $x$, every $e \in E(x)$ is such that $v(\alpha)_{x,e} = 0$. So $|\alpha|_x = \emptyset$, which means that $sv(\alpha)_x = 0$. □

**Proposition 4** *If* $\alpha \vDash_v \beta$*, then, for every* $x$, $sv(\alpha)_x \leq sv(\beta)_x$.

*Proof.* Assume that $\alpha \vDash_v \beta$ and consider any $x$. Since there is no $e \in E(x)$ such that $v(\alpha)_{x,e} = 1$ and $v(\beta)_{x,e} = 0$, we get that $|\alpha|_x \subseteq |\beta|_x$. By Definition 5 it follows that $sv(\alpha)_x \leq sv(\beta)_x$. □

Proposition 2 says that tautologies always get value 1. Proposition 3 says that contradictions always get value 0. Proposition 4 says that, when $\alpha \vDash_v \beta$, the value of $\beta$ cannot be lower than the value of $\alpha$. Note that, since $\alpha$ and $\beta$ are logically equivalent just in case $\alpha \vDash_v \beta$ and $\beta \vDash_v \alpha$, Proposition 4 entails that two logically equivalent formulas always have the same value.[7]

Second, according to $sv$, the value of a disjunction is definable in terms of the value of the corresponding conjunction.

**Proposition 5**

$$sv(\alpha \vee \beta)_x = sv(\alpha)_x + sv(\beta)_x - sv(\alpha \wedge \beta)_x$$

*Proof.* By Definition 5, $sv(\alpha \vee \beta)_x = \sum_{e \in |\alpha \vee \beta|_x} P_x(e)$. From this and Proposition 1 (b) we get that $sv(\alpha \vee \beta)_x = \sum_{e \in |\alpha|_x \cup |\beta|_x} P_x(e)$. Moreover, by Definition 5, $sv(\alpha)_x = \sum_{e \in |\alpha|_x} P_x(e)$, $sv(\beta)_x = \sum_{e \in |\beta|_x} P_x(e)$, and $sv(\alpha \wedge \beta)_x = \sum_{e \in |\alpha \wedge \beta|_x} P_x(e)$. The third conjunct and Proposition 1 (a) entail that $sv(\alpha \wedge \beta)_x = \sum_{e \in |\alpha|_x \cap |\beta|_x} P_x(e)$. Since $\sum_{e \in |\alpha|_x \cup |\beta|_x} P_x(e) = \sum_{e \in |\alpha|_x} P_x(e) + \sum_{e \in |\beta|_x} P_x(e) - \sum_{e \in |\alpha|_x \cap |\beta|_x} P_x(e)$, we get that $sv(\alpha \vee \beta)_x = sv(\alpha)_x + sv(\beta)_x - sv(\alpha \wedge \beta)_x$. □

Third, as results from the facts listed above, $sv$ is a probability function according to the standard definition:

**Proposition 6** *$sv$ satisfies the following constraints, for every $x$*:

(a) $sv(\alpha)_x \geq 0$;
(b) $sv(\alpha)_x = 1$ if $\alpha$ is logically true;
(c) $sv(\alpha \vee \beta)_x = sv(\alpha)_x + sv(\beta)_x$ if $\alpha \wedge \beta$ is logically false.

*Proof.* (a), or *non-negativity*, is implied by Definition 5. (b), or *normalization*, amounts to Proposition 2. (c), or *additivity* follows from Propositions 3 and 5. □

Note that, as a corollary of Proposition 6, we get that the value of the negation of a formula is a function of the value of the formula itself, that is, $sv(\neg\alpha)_x = 1 - sv(\alpha)_x$. To see why it suffices to note that $sv(\alpha \vee \neg\alpha)_x = 1$ by Proposition 2 and $sv(\alpha \wedge \neg\alpha)_x = 0$ by Proposition 3, so Proposition 6 (c) yields that $1 = sv(\alpha)_x + sv(\neg\alpha)_x$.

As in the case of the valuation level, one can define a consequence relation that holds at the supervaluation level. Here a plausible option is what Edgington calls the *constraining property*: for any valid argument, any assignment of probability to its premises and conclusion is such that the improbability of the conclusion does not exceed the sum of the improbabilities of the premises.[8] Assuming that improbability is expressed by a function $u$ such that $u(\alpha)_x = 1 - sv(\alpha)_x$, this relation, indicated as $\vDash_{sv}$, is defined as follows:

[7] In Kyburg (1970), an epistemic property along the lines of Proposition 4 is labelled 'weak deduction principle'.

[8] Edgington (1999), p. 300. This property was first identified in Adams (1966), with a different formulation.

**Definition 6** *$\alpha_1, \ldots, \alpha_n \vDash_{sv} \beta$ iff $u(\alpha_1)_x + \ldots + u(\alpha_n)_x \geq u(\beta)_x$ for every $x$ in every model.*

One way to see the plausibility of Definition 6 is to realize that it would make little sense to require that $sv(\alpha_i)_x \leq sv(\beta)_x$ for each $\alpha_i$ such that $1 \leq i \leq n$. For example, if $\beta = \alpha_1 \wedge \alpha_2$, it can happen that $sv(\alpha_1)_x > sv(\beta)_x$ and $sv(\alpha_2)_x > sv(\beta)_x$ even though $\alpha_1, \alpha_2 \vDash_v \beta$. This point emerges even more vividly when one considers paradoxical cases such as the lottery, where a plurality of highly probable premises leads to a clearly false conclusion. Suppose that 1000 lottery tickets are sold to 1000 persons $P_1, \ldots, P_{1000}$. For each $P_n$ such that $1 \leq n \leq 1000$, it is very likely that $P_n$ will not win. But it does not seem rational to accept the conjunction of the 1000 sentences so constructed, for that would amount to holding that nobody will win. According to Definition 6, the validity of an argument is compatible with the possibility that its conclusion has some degree of improbability that does not match the degree of improbability of any of its premises. So, in the case of the lottery it can be claimed that the reasoning is valid even though its conclusion is not probable at all.

A crucial equivalence result can be proved about the two notions of logical consequence defined for $v$ and $sv$ respectively:

**Proposition 7** *$\alpha_1, \ldots, \alpha_n \vDash_v \beta$ iff $\alpha_1, \ldots, \alpha_n \vDash_{sv} \beta$.*

*Proof.* To prove the left-to-right direction, assume that $\alpha_1, \ldots, \alpha_n \vDash_v \beta$. Then $\neg\beta \vDash_v \neg\alpha_1 \vee \ldots \vee \neg\alpha_n$. By Proposition 4 it follows that $sv(\neg\beta)_x \leq sv(\neg\alpha \vee \ldots \vee \neg\alpha_n)_x$. Moreover, $sv(\neg\alpha \vee \ldots \vee \neg\alpha_n)_x \leq sv(\neg\alpha_1)_x + \ldots + sv(\neg\alpha_n)_x$ because Proposition 5 entails that $sv(\gamma \vee \delta)_x \leq sv(\gamma)_x + sv(\delta)_x$, and this can be extended to any disjunction with $n$ disjuncts by grouping the first $n-1$ disjuncts into a single disjunct. So, $sv(\neg\beta)_x \leq sv(\neg\alpha_1)_x + \ldots + sv(\neg\alpha_n)_x$, which means that $1 - sv(\beta)_x \leq (1 - sv(\alpha_1)_x) + \ldots + (1 - sv(\alpha_n)_x)$, hence $u(\beta)_x \leq u(\alpha_1)_x + \ldots + u(\alpha_n)_x$.[9]

The right-to-left direction is proved by contraposition. Assume that $\alpha_1, \ldots, \alpha_n \nvDash_v \beta$. This means that in some model $\langle \mathcal{F}, P, V \rangle$ there is a pair $\langle x, e \rangle$ such that $v(\alpha_1)_{x,e} =, \ldots, = v(\alpha_n)_{x,e} = 1$ and $v(\beta)_{x,e} = 0$. Let $\langle \mathcal{F}, P, V' \rangle$ be a model such that, for every sentence letter $\gamma$ that occurs in the formulas $\alpha_1, \ldots, \alpha_n, \beta, V'(\gamma)_{x,e'} = V(\gamma)_{x,e}$ for every $e' \in E(x)$. In $\langle S, <, P, V' \rangle$ we thus get that, for every $e' \in E(x)$, $v(\alpha_1)_{x,e'} =, \ldots, = v(\alpha_n)_{x,e'} = 1$ and $v(\beta)_{x,e'} = 0$, which means that $sv(\alpha_1)_x =, \ldots, = sv(\alpha_n)_x = 1$ and $sv(\beta)_x = 0$. Therefore, $u(\alpha_1)_x =, \ldots, = u(\alpha_n)_x = 0$ and $sv(\beta)_x = 1$, hence $\alpha_1, \ldots, \alpha_n \nvDash_{sv} \beta$.

Proposition 7 shows that the consequence relation $\vDash_v$ defined at the valuation level and the consequence relation $\vDash_{sv}$ defined at the supervaluation level are extensionally equivalent. In this respect, quantitative supervaluationism is perfectly classical, which we take to be a desirable result. In the case of the lottery, for example, the fact that the argument turns out to be valid according to Definition 6 accords with the

[9] This proof follows Edgington (1999), p. 307.

standard assumption that a conjunction logically follows from the collection of its conjuncts.

## 4 Alethic interpretation: vagueness

So far we have outlined quantitative supervaluationism in purely formal terms, without addressing the question of how the values obtained at the supervaluation level are to be understood. Now we focus on the alethic interpretation of the semantics, the interpretation according to which $1$ stands for maximal truth, $0$ stands for minimal truth — that is, falsity — and every other number in the interval $[0, 1]$ indicates an intermediate degree of truth. This section shows how the alethic interpretation works in the case of vagueness.

According to Definition 1, a frame for L is a triple $\langle X, Y, E\rangle$, where $X$ is a set of points and $E$ fixes a set of extensions for each point in $X$. In order to deal with vagueness, points will be understood as *contexts*, and extensions will be understood as *precisifications*, that is, complete specifications of the meaning of sentences relative to contexts. As mentioned in Sect. 1, the underlying assumption here is that the vagueness of a sentence consists in its capacity of being made precise in more than one way.

According to Definition 2, a model of L is a triple $\langle \mathcal{F}, P, V\rangle$, where $\mathcal{F}$ is a frame, $P$ assigns to each point $x$ a proximity assignment $P_x$, and $V$ evaluates atomic formulas relative to point-extension pairs. $P_x$ is understood as a measure of *admissibility* for precisifications: a precisification of a sentence is admissible insofar as it is compatible with the meaning of the expressions that occur in the sentence. In the literature, admissibility is usually assumed to be a property that either belongs or does not belong to a precisification. But there seems to be nothing conceptually wrong with treating it as a gradable property, leaving room for the possibility that different precisifications have different degrees of admissibility.[10] Finally, the values of $V$ will be understood as truth values that simple sentences take in contexts relative to precisifications: to say that an atomic formula $\alpha$ has value 1, or 0, relative to $\langle x, e\rangle$ is to say that $\alpha$ is true, or false, in the context $x$ relative to the precisification $e$.

Once the models of L are so construed, Definitions 3 and 5 acquire the expected reading. Definition 3 specifies truth in a context relative to a precisification for any sentence, while Definition 5 yields the corresponding quantitative account of truth: the degree of truth of $\alpha$ in a context $x$ equals the total proximity value of the precisifications admissible in $x$ where $\alpha$ is true. Thus, maximal truth and minimal truth — that is, falsity — correspond to supertruth and superfalsity as defined in qualitative supervaluationism.[11]

[10] A proposal along these lines is made in Simons (2010), p. 485. Williams (2011) defines degrees of determinacy in terms of a simple count measure, namely in terms of the number of precisifications that make a sentence true. This is the kind of measure one gets in our framework when one restricts consideration to the cases in which $E(x)$ is finite and $P_x$ assigns the same value to every element of $E(x)$.

[11] The resulting notion of truth is what Simons (2010) would call “expected truth value”.

In order to see the appeal of alethic quantitative supervaluationism as applied to vagueness, it is useful to compare it with truth-functional continuum valued logic, which rests on the idea that sentential connectives must satisfy generalizations of truth-functionality to degrees of truth. In particular, the degree of truth of a conjunction is defined as the minimum of the degrees of truth of its conjuncts, and the degree of truth of a disjunction is defined as the maximum of the degrees of truth of its disjuncts.[12]

To illustrate the difference, we use two examples drawn from Williamson's discussion of continuum valued logic. The first example is about conjunction. If $p$ is true to the same degree as $q$, by generalized truth-functionality it follows that $p \wedge q$ is true to the same degree as $p \wedge p$, which in turn is the same degree as $p$. Now imagine someone drifting off to sleep, and consider the following sentences:

(1) He is awake
(2) He is asleep

At some point (1) and (2) will have the same degree of truth, an intermediate one. So the following conjunction will have the same intermediate degree of truth:

(3) He is awake and he is asleep

But this is highly implausible, since waking and sleep by definition exclude each other. Intuitively, (3) has no chance of being true.[13]

The second example is about disjunction. If $p$ is true to the same degree as $q$, it follows by generalized truth-functionality that $p \vee q$ is true to the same degree as $p \vee p$, and therefore as $p$. Thus if (1) and (2) have the same intermediate degree of truth, so has the following sentence:

(4) He is awake or he is asleep

But again, this is highly implausible, given that intuitively the degree of truth of (4) should be higher.[14]

Alethic quantitative supervaluationism differs from continuum valued logic with respect to these two examples because it treats degrees of truth as probabilities, as explained in Sect. 3. Even supposing that (1) and (2) have exactly the same intermediate degree of truth, it does not follow that (3) has that degree of truth. The degree of truth of (3) is considerably lower, and may be 0 on the plausible assumption that (3) is false in all admissible precisifications. Similarly, from the supposition that (1) and (2) have exactly the same intermediate degree of truth, it does not follow that (4) has

[12] The idea goes back to Łukasiewicz, see Łukasiewicz and Tarski (1930). Here we will not provide a detailed discussion of the most recent developments of the debate on degrees of truth. Smith (2008) provides a critical overview.

[13] Williamson (1994), p. 136.

[14] Williamson (1994), p. 137.

that degree of truth. The degree of truth of (4) is considerably higher, and may be 1 on the plausible assumption that (4) is true in all admissible precisifications.

More generally, alethic quantitative supervaluationism is immune to any objection that appeals to the apparent failure of generalized truth-functionality. Arguably, degrees of truth need not be associated with the idea that sentential connectives must preserve truth-functionality. As Edgington pointed out, one obtains more plausible results if one assumes that degrees of truth behave as probabilities, which is exactly what we get in alethic quantitative supervaluationism. In particular, given the constraining property — Proposition 7 — the sorites paradox may coherently be treated in analogy with the lottery as a valid argument formed by a series of almost perfectly true premises and a false conclusion.[15]

Are there objections that specifically affect alethic quantitative supervaluationism as distinct from continuum valued logic? Williamson thinks that there are. He considers and rejects the hypothesis of supervaluational degrees of truth in connection with comparative adjectives. More precisely, following Lewis and Kamp, he phrases this hypothesis in terms of the assumption that $a$ is $F$er than $b$ if and only if '$a$ is $F$' is truer than '$b$ is $F$'. His objections depend precisely on this assumption. One is this:

> Since the truth of 'David is braver than Saul' requires 'David is brave' to be truer than 'Saul is brave', it is incompatible with the truth of 'Saul is brave'. Thus 'Saul is brave, but David is braver than Saul' cannot be true. That is absurd. The brave are not all equally brave.[16]

Another objection in a similar vein is this:

> Consider 'acute' as an adjective of angles. It is precise, for '$a$ is acute' is true if $a$ is less than a right angle, and false otherwise. 'An angle of $60°$ is acute' is true, and therefore true on every admissible valuation. Nevertheless, an angle of $30°$ is more acute than an angle of $60°$.[17]

In the first case Williamson takes the target theory to imply that, if David is braver than Saul, 'David is brave' is truer than 'Saul is brave'. In the second he takes the target theory to imply that, if an angle of $30°$ is more acute than an angle of $60°$, 'An angle of $30°$ is acute' is truer than 'An angle of $60°$ is acute'.

These objections, however, overlook a crucial fact: the hypothesis of supervaluational degrees of truth does not imply that $a$ is $F$er than $b$ if and only if '$a$ is $F$' is truer than '$b$ is $F$', and it is not clear why one should accept this equivalence. In particular, the left-to-right direction is far from obvious: although it may be plausible to assume that if '$a$ is $F$' is truer than '$b$ is $F$', then $a$ is $F$er than $b$, the converse does not seem to hold. In a supervaluationist framework, it is reasonable to leave room for the possibility that $a$ and $b$ are both clear cases of $F$ — in that both of them are $F$ in all admissible precisifications — in spite of the fact that $a$ is $F$er than $b$. For exam-

[15] Edgington (1992), pp. 200–201, Edgington (1999), pp. 308–309.

[16] Williamson (1994), p. 156.

[17] Williamson (1994), p. 156.

ple, consider a soritical series of persons $P_{2.00}, P_{1.99}, \ldots, P_{1.00}$ such that $P_{2.00}$ is 2 meters tall, $P_{1.99}$ is 1.99 meters tall, and so on. In this case, the theory should imply that both $P_{2.00}$ and $P_{1.99}$ are clear cases of tallness, so they are tall in all admissible precisifications, in spite of the fact that $P_{2.00}$ is taller than $P_{1.99}$. But if the left-to-right direction of the biconditional fails, the same goes for Williamson's objections. From the premise that David is braver than Saul one cannot infer that 'David is brave' is truer than 'Saul is brave', hence it is perfectly fine to say 'Saul is brave, but David is braver than Saul'. Similarly, from the premise that an angle of $30°$ is more acute than an angle of $60°$, one cannot infer that 'An angle of $30°$ is acute' is truer than 'An angle of $60°$ is acute', so the two sentences can be equally true.

## 5 Alethic interpretation: future contingents

To see how alethic quantitative supervaluationism can be applied to future contingents, let us start again from the basic ingredients of the semantics. According to Definition 1, a frame for L is a triple $\langle X, Y, E \rangle$, where $X$ is a set of points and $E$ fixes a set of extensions for each point in $X$. In this case, points are understood as *moments*, that is, as minimal temporal units, and extensions are understood as *histories*, that is, possible courses of events. So, each moment $x$ will be associated with a set of histories $E(x)$. In standard branching time structures, the histories associated with $x$ are represented by lines that go through $x$. The underlying assumption is that, for any time at which a sentence may be uttered, there is a plurality of possible futures compatible with the way things are at that time.[18]

According to Definition 2, a model of L is obtained by adding to a frame a proximity function $P$ and a valuation function $V$. Here each $P_x$ will be understood as a measure of *accessibility* for histories: a history $e$ is accessible at a moment $x$ insofar as it constitutes a real possibility from the point of view of $x$. In standard modal semantics, accessibility is assumed to be a relation that either obtains or does not obtain between worlds. But there seems to be nothing conceptually wrong with treating it as a gradable relation, leaving room for the possibility that different histories have different degrees of accessibility. Finally, the values of $V$ will be understood as truth values that simple sentences take at moments relative to histories: to say that an atomic formula $\alpha$ has value 1, or 0, relative to $\langle x, e \rangle$ is to say that $\alpha$ is true, or false, at the moment $x$ relative to the history $e$.

On this interpretation of the models of L, Definition 3 specifies truth at a moment relative to a history for any sentence. Note that L does not contain temporal operators, so tensed sentences are formalized in L by using simple propositional formulas. It is easy to see how L could be enriched by adding such operators. In particular, a metric future operator $F_n$ could be defined as follows: $F_n\alpha$ is true at $\langle x, e \rangle$ just in case $\alpha$ is true at $\langle y, e \rangle$, where $y$ lies at $n$ units after $x$ in $e$. Similarly, a non-metric future operator $F$ could be defined as follows: $F\alpha$ is true at $\langle x, e \rangle$ just in case $\alpha$ is true at $\langle y, e \rangle$ for some $y$ after $x$ in $e$. But since it is not essential for our purposes to

[18] Within our framework, branching time structures can easily be obtained by imposing appropriate constraints on the frame.

rely on such enrichment, we will simply assume that a future-tense sentence is true at a moment relative to a history just in case the history makes true what the sentence says, possibly in virtue of facts or events located at later moments.

Definition 5 yields the corresponding quantitative account of truth: the degree of truth of $\alpha$ at a moment $x$ equals the total proximity value of the histories accessible at $x$ where $\alpha$ is true. Thus, maximal truth and minimal truth — that is, falsity — correspond to supertruth and superfalsity as understood in qualitative supervaluationism.

As in the case of vagueness, alethic quantitative supervaluationism can be compared with continuum valued logic, although the latter is admittedly less relevant to the current debate on future contingents. The contrast emerges with clarity if one considers examples that are structurally similar to those discussed in Sect. 4. Imagine that some sort of precipitation is about to occur but there is no way to say whether the temperature will be above $0^\circ$ by then. It is easy to see that, in this scenario, the following sentences give rise to problems similar to those illustrated by (1)–(4):

(5) It will rain
(6) It will snow
(7) It will rain and it will snow
(8) Either it will rain or it will snow

Alethic quantitative supervaluationism is immune to such problems, as it implies that degrees of truth behave as probabilities.

From now on, following Edgington, we will call *verity* the degree of truth of a sentence, in order to keep track of its probabilistic structure and avoid confusion with supertruth as defined in qualitative supervaluationism.[19] In particular, we will take for granted that maximal verity, the value 1, amounts to perfect truth, while minimal verity, the value 0, amounts to perfect falsity. Alethic quantitative supervaluationism, both in the case of vagueness and in the case of future contingents, provides an analysis of truth in terms of verity.

A final remark on the notion of verity is in order. So far we have shown that alethic quantitative supervaluationism is free from some troubles that affect previous attempts to model degrees of truth in a formal semantics. This by itself does not justify an analysis of truth in terms of verity. It might be argued that there are purely conceptual reasons for thinking that truth does *not* admit degrees. Whether or not there are such reasons, however, is a substantive question that goes far beyond the scope of our investigation. All we can say here is that, if there is nothing conceptually wrong in the hypothesis that truth admits degrees, alethic quantitative supervaluationism provides a coherent way to spell out this hypothesis.[20]

[19] The term 'verity' is introduced in Edgington (1999). We will not use the plural 'verities', though, because we don't like it.

[20] This is not quite the same thing as to say that it is the only way. For example, Douven and Decock (2017) suggests a different account of verity.

## 6 Epistemic interpretation: vagueness

In the alethic interpretation, the property defined through our quantitative method is understood as a measure of representational adequacy: the verity of a sentence indicates how adequate is the sentence as a representation of reality. So it is not an epistemic property. Assuming that representational adequacy is a matter of what the sentence says and of the way things are, it is independent of any consideration about the justification that one may have for asserting the sentence. However, there is an alternative interpretation according to which the property defined is epistemic. As we will suggest, this alternative interpretation has some interesting applications that have not been fully appreciated so far.

In order to distinguish the epistemic interpretation from the alethic interpretation, we call *credibility* the epistemic property that pertains to the former. The credibility of a sentence amounts to the degree of acceptance that one should rationally assign to the sentence. So it is not to be identified with credence, understood as subjective degree of belief, although the difference between the two properties will not emerge at the formal level, and most of what we will say about the epistemic interpretation can equally be phrased in terms of credence. All that matters for our purposes is that credibility is assumed to be distinct from verity: in the most extreme cases, a sentence can be highly credible but perfectly false, just as it can be little credible but perfectly true.

In Sect. 4 we saw how alethic quantitative supervaluationism applies to vagueness: the verity of a sentence is defined in terms of the total proximity value of the admissible precisifications in which the sentence is true. Epistemic quantitative supervaluationism can be phrased in a similar way. Instead of assuming that precisifications are actually admissible, one can assume that they are epistemically admissible, that is, that they are reasonably believed to be compatible with the meaning of the expressions that occur in the sentence. If 'admissible' is so construed, the credibility of a sentence is defined in terms of the total proximity value of the admissible precisifications in which the sentence is true.

The alethic interpretation and the epistemic interpretation thus provide two structurally identical but extensionally distinct dimensions of variation. The credibility of a sentence in a given context may differ from its verity in that context, even though formally speaking the two values are obtained in exactly the same way. In order for the credibility of a sentence in a given context to be identical to its verity in that context, there should be perfect match between the precisifications that are epistemically admissible in that context and those that are actually admissible in that context. But it cannot be taken for granted that there is such a match.

In any case, the alethic interpretation and the epistemic interpretation can in principle be combined in a two-dimensional account of vagueness where verity and credibility are defined as distinct properties, possibly using distinct proximity functions.[21]

[21] Although some objections have been raised against the probabilistic treatment of degrees of belief in the case of vagueness, see Schiffer (2000), it is not immediately obvious that such objections apply to a two-dimensional account of the kind envisaged, given that credibility as understood here is not quite the same thing as degree of belief.

The specificity of epistemic quantitative supervaluationism as applied to vagueness emerges even more clearly when truth is *not* equated with verity. To illustrate this point, we consider two non-supervaluationist views of vagueness that are equally compatible with epistemic quantitative supervaluationism.

The first view, *epistemicism*, describes vagueness as a form of ignorance: although the boundaries of our concepts are sharp, we do not know exactly where those boundaries lie, and this is why in some cases it is unclear whether certain predicates apply to certain objects. Recall the soritical series considered in Sect. 4, where $P_{2.00}, P_{1.99}, \ldots, P_{1.00}$ are such that $P_{2.00}$ is 2 meters tall, $P_{1.99}$ is 1.99 meters tall, and so on. Epistemicism implies that, for every person $P_i$ in the series, there is a fact of the matter whether $P_i$ belongs to the extension of 'tall', so '$P_i$ is tall' is either true or false. This view, initially suggested by Campbell and Sorensen, has been thoroughly spelled out and defended by Williamson.[22]

The second view, *nihilism*, rests on the idea that truth and falsity require precision, so a vague sentence — a sentence that contains vague expressions — cannot be true or false. Such a sentence does not express a proposition in the relevant sense, that is, a content that is evaluable as true or false. For example, for each person $P_i$ in the series mentioned above, '$P_i$ is tall' does not express such a content. This view, which goes back to Frege, has been elaborated in different ways by Ludwig and Ray, Braun and Sider, and Iacona.[23]

The contrast between epistemicism and nihilism can be phrased by using the notion of precisification. Arguably, both views agree that if a sentence is vague, there is a set of admissible precisifications such that the sentence has a definite truth value relative to each of them. The key difference is the following. According to epistemicism, one of the precisifications in the set is correct in some absolute sense, as it assigns to the sentence its real truth conditions. So, the sentence is true *simpliciter* just in case it is true relative to that precisification. According to nihilism, instead, there is no such thing as the correct precisification. All the precisifications in the set are equal, so to say, and there is no intelligible notion of truth *simpliciter* over and above truth relative to precisifications.[24]

Although what has been just said holds no matter how precisifications are understood, here we will focus on the epistemic understanding. On the assumption that a vague sentence has a set of epistemically admissible precisifications, its credibility can be defined in accordance to quantitative supervaluationism. The definition is compatible both with epistemicism and with nihilism. If one endorses epistemicism, one will say that the sentence, in addition to being credible to some extent, is either true or false. If one endorses nihilism, one will deny that the sentence has a truth value in addition to its credibility value. In both cases, credibility and truth turn out to be distinct and independent properties.

[22] The version of epistemicism suggested in Campbell (1974) and Sorensen (1988) may be regarded as ontic, while the one advocated by Williamson (1994) is distinctively semantic.

[23] Frege 1903, p. 168, claims that truth and falsity require precision. Ludwig and Ray (2002), Braun and Sider (2007), Iacona (2010), and Iacona (2024) provide different versions of nihilism.

[24] This is the characterization of nihilism suggested in Iacona (2024).

## 7 Epistemic interpretation: future contingents

Epistemic quantitative supervaluationism has another interesting application, which concerns future contingents. As we saw in Sect. 5, when alethic quantitative supervaluationism is adopted as an account of future contingents, the verity of a sentence is defined in terms of the total proximity value of the accessible histories in which the sentence is true. The epistemic interpretation differs from the alethic interpretation in that accessibility is understood epistemically: the accessible histories are construed as the courses of events that are rationally believed to be compatible with the way things presently are. The credibility of a sentence can thus be defined in the same way as the total proximity value of the accessible histories in which the sentence is true.[25]

As in the case of vagueness, one can in principle develop a two-dimensional account of future contingents, treating verity and credibility as structurally identical but extensionally distinct properties. But it is only in combination with a non-supervaluationist view on future contingents that one can fully appreciate the specificity of epistemic quantitative supervaluationism. To illustrate this point, we consider two views of future contingents that are structurally similar to the two views of vagueness considered in Sect. 6.

The first view, *Ockhamism*, provides an analysis of truth in terms of future actuality. On this view, a future contingent $\alpha$ uttered at a moment $x$ is to be understood as a description of the actual course of events, which is one among the many courses of events that are possible at $x$. So the truth or falsity of $\alpha$ depends on what happens in that particular course of events. For example, in the situation described in Sect. 5, the truth or falsity of (5) depends on whether it will actually rain. As shown by Øhrstrøm, Rosenkranz, Iacona, Cariani and Santorio, Malpass and Wawer, among others, this idea can be formally articulated by defining the truth value of $\alpha$ at $x$ as the value that $\alpha$ takes at $x$ relative to the actual history.[26]

The second view, advocated by Belnap and others, implies a form of relativism about truth: future contingents lack truth values because it is only relative to histories that they express truth evaluable contents. On this view, which may be called *history-relativism*, when a future contingent $\alpha$ is uttered at a moment $x$, the description it provides is incomplete, so to say, due to the fact that a plurality of courses of events are possible at $x$. So $\alpha$ is evaluable as true or false only relative to this or that possible course of events. For example, (5) is true relative to some histories and false relative to others, but does not have any absolute truth value.[27]

The contrast between Ockhamism and history-relativism is structurally similar to the contrast between epistemicism and nihilism. Both views agree that, for any future contingent $\alpha$ and any moment $x$, there is a set of accessible histories relative to which $\alpha$ has a definite truth value. However, they crucially differ as to whether $\alpha$ is also true or false in some absolute sense. Ockhamism maintains that one of the

[25] This definition of credibility has been explored in Iacona and Iaquinto (2021), where rational belief is understood in terms of objective chance.

[26] Øhrstrøm (2009), Rosenkranz (2012), Iacona (2014), Cariani and Santorio (2018), Malpass and Wawer (2020).

[27] Belnap et al. (2001).

accessible histories is the history to which $\alpha$ naturally refers. When $\alpha$ is true relative to that history, it is true *simpliciter*, otherwise, it is false *simpliciter*. On the contrary, history-relativism regards all accessible histories as equal, for $\alpha$ singles out no particular history over the others. Thus, there is no meaningful sense in which $\alpha$ can be true or false *simpliciter*.

Epistemic quantitative supervaluationism can be combined with Ockhamism and with history-relativism. If Ockhamism is adopted, future contingents will be assigned both a credibility value and a truth value. It is important to note that, within this framework, a future contingent can have a high degree of credibility despite being false, or a low degree of credibility despite being true. If history-relativism is adopted, instead, future contingents will only be assigned a degree of credibility, as they are considered neither true nor false. Regardless of the view adopted, credibility will vary independently of how or whether future contingents are assigned a truth value.

## 8 Quantitative vs qualitative

The foregoing sections spell out quantitative supervaluationism and explain how it can be employed to address key issues such as vagueness and future contingents. This section compares quantitative supervaluationism with qualitative supervaluationism in order to point out some theoretical advantages of the former over the latter.

First of all, quantitative supervaluationism may be regarded as a generalization of qualitative supervaluationism, in that the latter is definable in terms of the former. To see why it suffices to think that qualitative supervaluationism is obtained by replacing definition 5 with a coarser-grained definition that yields just three values: 1, 0, and 0.5, where the latter is assigned whenever definition 5 delivers a value other than 1 or 0. In other words, qualitative supervaluationism is obtained by modifying definition 5 so that $sv(\alpha)_x = 0.5$ whenever $\sum_{e \in |\alpha|_x} P_x(e)$ is neither 1 nor 0. It is easy to see that, when one restricts consideration to trivalent assignments of this kind, the formal properties of quantitative supervaluationism — as expressed by propositions 1–7 — still hold.

Insofar as quantitative supervaluationism includes the trivalent assignments just considered as a special case, its relation with qualitative supervaluationis is somehow analogous to the relation between continuum valued logic and three-valued logic. This analogy emerges clearly when one considers the treatment of conjunction and disjunction. As explained in Sects. 4 and 5, quantitative supervaluationism differs from continuum valued logic in that it leaves room for the possibility that the value of a conjunction is lower than the values of its conjuncts, and the value of a disjunction is higher than the values of its disjuncts. This is exactly the way in which qualitative supervaluationism differs from three-valued logic.

Further theoretical virtues of quantitative supervaluationism concern the two interpretations illustrated in Sects. 4–7. Let us start with the alethic interpretation. As observed in Sect. 5, the question whether truth admits degrees is a substantive question that cannot be addressed here. So it would make no sense to defend quantitative supervaluationism as linguistically or metaphysically superior to qualitative supervaluationism. Nonetheless, there might be theoretical contexts in which a quantitative approach yields more interesting results than a qualitative one. At least two examples can be used to illustrate such a possibility.

The first example is the issue of higher-order vagueness, which has been raised in connection with supervaluationism in its traditional qualitative formulation. According to that formulation, a supervaluation sharply divides the sentences of a language into three classes: true, false, and neither true nor false. This is at odds with the fact that there is no clear distinction between the cases in which a vague expression clearly applies, or does not apply, and those in which it is unclear whether it applies. Williamson phrases the problem as follows:

> Supervaluationists often regard admissibility as consistency with the semantic rules of the language. If the rules decide a case, then an admissible interpretation decides it in the same way; it may decide a case when they do not. Since consistency is a matter of logic, admissibility looks as though it should be a precise concept. Higher-order vagueness shows this picture to be misleading.[28]

It is certainly correct to point out that, if admissibility is understood as a precise qualitative notion, supervaluationism fails to account for higher-order vagueness. However, it is not essential to supervaluationism that admissibility is understood that way. One can coherently recognize that admissibility is a vague notion, and think that the best way to make sense of it in formal terms is through a quantitative measure of the sort suggested here. This is not to say that quantitative supervaluationism solves the problem of higher-order vagueness. Proximity assignments as defined above are precise, so the question remains of how they relate to their natural language counterparts. But at least they might provide a better formal model of the notion of admissibility.

The second example concerns nihilism. As we have seen, nihilism holds that ordinary sentences are not evaluable as true or false because they are not precise. So, a main challenge for nihilism is to explain why speakers normally take ordinary sentences to be true or false. Braun and Sider suggest an explanation in terms of *approximate truth*: speakers typically ignore vagueness, and they can safely do so insofar as the sentences they use are approximately true. Approximate truth is then defined in the same way in which qualitative supervaluationism defines supertruth.[29] However, even granting that approximate truth is definable in terms of truth relative to legitimate disambiguations, as they suggest, it is still not obvious that approximate truth should be defined in qualitative terms as truth in all legitimate disambiguations. After all, approximation is reasonably understood as a gradable notion, and it is quite natural to think that there are correct comparative judgments concerning closeness to truth. Consider the following sentences:

(9) $P_{1.90}$ is tall
(10) $P_{1.75}$ is tall

[28] Williamson (1994), p. 158.

[29] Braun and Sider (2007), p. 135.

It is plausible to expect that (9) is closer to truth than (10), even assuming that neither of them is literally true. So, perhaps a quantitative definition of approximate truth along the lines suggested here would be a better option for the nihilist.[30]

Now let us turn to the epistemic interpretation. When it comes to this interpretation, quantitative supervaluationism clearly stands out as a better option if compared with qualitative supervaluationism. Consider the case of vagueness. Independently of how truth values are to be assigned, sentences containing vague expressions are not all alike from the epistemic point of view. For example, (9) and (10) are not equally compelling. It is definitely more reasonable to accept (9) than to accept (10). Even when considering two adjacent items in a soritical series, comparative judgments of acceptability typically exhibit a detectable asymmetry. Consider the following sentence:

(11) $P_{1.74}$ is tall

Although there is a plausible sense in which (10) is slightly more reasonable than (11), there is no plausible sense in which (11) is more reasonable than (10). This is also shown by the intuitive contrast between the following sentences:

(12) $P_{1.75}$ is tall and $P_{1.74}$ is not tall
(13) $P_{1.74}$ is tall and $P_{1.75}$ is not tall

No matter how bad (12) may look, (13) seems to be unacceptable in a way in which (12) is not.

Quantitative supervaluationism provides a straightforward explanation of these intuitive differences, for one can assign degrees of credibility to (9)–(13) in accordance with definition 5, independently of whether epistemicism, nihilism, or any other view of vagueness is adopted. The credibility of (9) turns out to be considerably higher than the credibility of (10). The credibility of (10) turns out to be slightly higher than the credibility of (11). Moreover, while (12) has a positive degree of credibility, although very low, (13) is not credible at all, as no admissible precisification makes it true. An explanation along these lines is not available to qualitative supervaluationism, which can only provide a non-gradable notion of credibility.

Similar considerations hold for future contingents. Independently of how truth values are to be assigned, future contingents are not all alike from the epistemic point of view. Here is an example:

(14) The sun will rise tomorrow.
(15) The sun will not rise tomorrow.
(16) The coin will land head.
(17) The coin will land tails.

[30] This point is made in Iacona (2024). Note that the same kind of definition could be adopted in the case of history-relativism, given the analogy between nihilism and history-relativism pointed out in Sect. 7. History-relativism faces a similar explanatory challenge, for speakers ordinarily describe future contingents as being true or false.

Clearly, (14) differs from (15) in a way in which (16) does not differ from (17): it is more reasonable to believe (14) than to believe (15), whereas (16) and (17) should be assigned the same degree of belief. Again, quantitative supervaluationism accurately captures this difference in a way in which qualitative supervaluationism cannot, as the latter is unable to treat credibility as a gradable property. Thus, independently of whether one adopts Ockhamism, history-relativism, or any other view of future contingents, quantitative supervaluationism is able to vindicate the crucial distinction between what is true about the future and what is reasonable to believe about the future.

## 9 Assertibility

As we have seen, quantitative supervaluationism admits an alethic interpretation and an epistemic interpretation. The alethic interpretation has remarkable theoretical virtues, and yields an account of approximate truth that suits at least some views of vagueness and future contingents, such as nihilism and history-relativism. But it is the epistemic interpretation that offers the strongest reasons in favor of quantitative supervaluationism. Independently of one's preferred theory of vagueness or future contingents, quantitative supervaluationism vindicates an intuitive distinction that its qualitative counterpart is unable to capture, the distinction between truth and epistemic notions such as rational acceptability.

In order to appreciate the importance of this point, it is useful to think about assertibility, which is usually understood as an epistemic property of sentences distinct from truth. Although assertibility is sometimes described as a non-gradable property, it is quite common to treat it as a gradable property that somehow measures the evidence, justification, or entitlement that one may have for making a statement. In the second case, it is quite reasonable to assume that there is a direct relation between assertibility and credibility: a sentence is assertible to the extent that it is credible. As Lewis once put it,

> The truthful speaker wants not to assert falsehoods, wherefore he is willing to assert only what he takes to be very probably true.[31]

Considerations about assertibility are clearly relevant in the case of vagueness. For example, the observations made in Sect. 8 about (9)–(13) can easily be rephrased in terms of assertibility. (9) is definitely more assertible than (10), and there is a clear asymmetry between (10) and (11) when it comes to comparative judgments of assertibility, as is shown by the fact that (12) and (13) are not equally assertible. If the assertibility of a sentence containing vague expressions is defined in terms of its credibility, it can be claimed in accordance with our semantics that the assertibility of (9) is considerably higher than the assertibility of (10), that the assertibility of (10) is slightly higher than the assertibility of (11), that (12) has a very low degree of assert-

[31] D. D. Lewis (1976), p. 279.

ibility, and that (13) not assertible at all. This holds independently of the truth values that one's favourite view of vagueness ascribes to (9)-(13).

The case of future contingents is similar. For example, (14) is intuitively more assertible than (15), while the same does not hold for (16) and (17), even though no principled distinction can be drawn between the two pairs of sentences in terms of truth values. This explanatory problem, which is orthogonal to the main logical and metaphysical issues concerning future contingents, is known in the literature as the "assertion problem".[32] If the assertibility of a future contingent is defined in terms of its credibility, the assertion problem can be addressed by using our semantics. In particular, it can be claimed that the assertibility of (14) is higher than the assertibility of (15), while the assertibility of (16) and (17) is exactly the same. Again, this holds independently of the truth values that one's favourite view of future contingents ascribes to (14)–(17).

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## Declarations

**Conflict of interest** The authors declare no conflicts of interest.



[32] The label is due to Belnap et al. (2001). For recent discussions, see Malpass and Wawer (2012), Green (2014), Stojanovic (2014), MacFarlane (2014), Hattiangadi and Besson (2014), Santelli (2021).

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