



Kant's third law of mechanics: The long shadow of Leibniz

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ABSTRACT

This paper examines the origin, range and meaning of the Principle of Action and Reaction in Kant's mechanics. On the received view, it is a version of Newton's Third Law. I argue that Kant meant his principle as foundation for a Leibnizian mechanics. To find a 'Newtonian' law of action and reaction, we must look to Kant's 'dynamics,' or theory of matter.

I begin, in part I, by noting marked differences between Newton's and Kant's laws of action and reaction. I argue that these are explainable by Kant's allegiance to a Leibnizian mechanics. I show (in part II) that Leibniz too had a model of action and reaction, at odds with Newton's. Then I reconstruct how Jakob Hermann and Christian Wolff received Leibniz's model. I present (in Part III) Kant's early law of action and reaction for mechanics. I show that he devised it so as to solve extant problems in the Hermann-Wolff account. I reconstruct Kant's views on 'mechanical' action and reaction in the 1780s, and highlight strong continuities with his earlier, pre-Critical stance. I use these continuities, and Kant's earlier engagement with post-Leibnizians, to explain the un-Newtonian features of his law of action and reaction.

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A wide consensus, spanning over a century, has it that Kant's metaphysics of nature in the 1780s lays ground for Newtonian science. This claim has several guises, but all urge that, to make good sense of his *Metaphysical Foundations of Natural Science* (*MAN*), we must turn to Newton.

In this paper, I argue against one version of this idea. I examine the law of action and reaction in Kant's *mechanics*, and claim it was not meant to support Newton's mechanics. Instead, we should look to Leibniz and his successors to uncover its meaning and role in Kant. It turns out that he offers this law so as to solve problems in post-Leibnizian dynamics. This finding, I suggest, calls on us to rethink the basis and scope of Kant's a priori mechanics; it also warrants a reevaluation of Leibniz's legacy for natural philosophy in the Age of Reason.

1. Kant and 'Newtonian' science

In the 1880s, a group of Kant scholars began to assert that his metaphysics of body is best read in the context of the rising

'Newtonian' science of its time. Though rather vague, the claim seduced enough to survive into this century, despite sporadic doubts.¹ On a closer look, the alleged link between Kant and Newton turns out to have several strands, but three stand out for my topic. (1) The strongest is an ingenious construal that Michael Friedman laid down in exquisite detail; for him, Kant's *MAN* shows that key Newtonian concepts—absolute space, true motion, and universal gravitation—need synthetic a priori principles that Kant first specifies and defends. These concepts loom large in Newton's *Principia*, but rest there on a metaphysics unacceptable to Kant. In its place, he offers the principles of transcendental idealism, which he further specifies in *MAN*, so as to ground Newton's gravitation theory and doctrine of true motion.² The Friedman thesis assumes that Kant and Newton work with the same laws of motion.³ (2) Some have argued for the weaker view that Kant shows the three laws of Newton's mechanics to be derivable from resources in the First Critique. Kant's Analogies of Experience and analysis of the concept of matter—so goes the claim—yield Newton's laws, thus shown to be synthetic a priori. This view has a long lineage. It began in Germany

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¹ See, e.g., Adickes (1924) and Schäfer (1966). All translations are mine, unless noted.

² Cf. Friedman (1992), pp. 136–164.

³ For him, Kant's Phenomenology "outlines a procedure for applying [Kant and Newton's] laws of motion... so as to subject the given appearances (viz., apparent motions) to the modal categories in three steps or stages."—Friedman (1994), p. 33; my emphasis.

around 1885, and leading interpreters still defend it in our time. As a nod to its revered descent, let us call it the ‘Marburg reading.’⁴ (3) Finally, some take Newtonian science to be classical mechanics.⁵ Here, ‘classical’ is opposed to ‘relativistic.’⁶ In this sense, the claim is true: Kant’s *MAN* does ground *a* mechanics, and it is classical. But to associate it with Newton is misleading; in classical mechanics, the attribute ‘Newtonian’ is honorific, not descriptive. Newton voiced (or took for granted) what his foes also believed unshakably: Galilean kinematics. Their quibbles with Newton over space, time and motion do not make their own mechanics any less classical. Also, Kant grounds *classical* mechanics where few expect it—in his Phoronomy, not the oft-read Dynamics, Mechanics or Phenomenology. Lastly, this view takes some liberties with history: classical mechanics has *two* versions, with Newton to credit (in part) for one, not both. The other comes from Lagrange and others.

In this paper, I argue against (2). I explain how Newton’s and Kant’s laws of motion diverge, and claim it is because Kant meant his third ‘mechanical’ law to solve some extant problems in post-Leibnizian dynamics (Sections 1.1 and 1.2). Then I document my claim. I show that Leibniz too had a law of action and reaction (Section 2), and I recount its fate in the works of two disciples, Jakob Hermann (Section 2.1) and Christian Wolff (Section 2.2 and 2.3). Next, I show that Kant’s early law of action and reaction corrects the Hermann-Wolff Reaction Principle (Section 3), and spell out its continuities with his Critical model of mechanical interaction (Section 3.1). I conclude with a brief discussion of the limits of Kant’s *a priori* mechanics (Section 3.2), and a call for a more nuanced understanding of his foundations for physics—one that makes room for Leibniz’s legacy as well.⁷

Lastly, I should make very clear that, in this paper, I am only after the third law of Kant’s *mechanics*, i.e. his law of “the action of moving bodies on one another by communication of motion,” as he puts it. Though he has a ‘dynamical’ law of action and reaction, I make no attempt to investigate its content or range; nor do I mean to deny that it is compatible with Newton’s thought. In fact, I agree with scholars who claim that it can support action-at-a-distance cases such as mutual orbiting.

1.1. Newton and Kant on the laws of motion

I must begin with a closer look at the exact content and range of Newton’s and Kant’s laws of mechanics.⁸ Early in his *opus*, Newton states three “axioms, or laws of motion.” First is the Law of Inertia: a body continues with constant velocity unless an unbalanced *impressed* force acts on it.⁹ The second measures an impressed force: it equals the “change in motion” of the body *acted upon*.¹⁰ The third

reveals that impressed forces come in pairs: when a force acts upon a body, the body acts back upon the source of that force. “To any action there is always an opposite and equal reaction; in other words, the actions of two bodies upon each other are always equal and always opposite in direction.”

Compare these with Kant’s laws. (K1) The first asserts conservation of mass throughout corporeal changes.¹¹ (K2) Next is a law of inertia: “All change in matter has an external cause. A body persists in its state (of rest or uniform motion in the same direction) unless compelled by an external cause to leave this state.” (K3) Kant also has a Reaction Principle: “In all communication of motion, action and reaction are always equal to each other.” Plainly, they do not map directly onto Newton’s laws. Also, only two are proper laws of *motion*; the first entails nothing about the motion of bodies. So it seems hard to accept that Kant justifies Newtonian science by deriving Newton’s laws of motion. Note a glaring lacuna: the key to Newton’s mechanics—*impressed force* and the Second Law—is visibly absent from Kant’s *a priori* foundations.

Some interpreters noticed this, and have argued astutely that (K2) and (K3) are equivalent to Newton’s three laws. Yet accounts of this alleged equivalence have changed over time, a sign that it is not direct. At first, they claimed that Kant’s K2 is equivalent to Newton’s First and Second Laws.¹² But that needed emendation. If anything, Kant’s law of inertia says *less* than Newton’s similar law. With *Lex Prima*, Newton also codifies that only impressed force changes inertial states, not just any cause. However, Kant’s law says merely that any change in a body’s state is caused by external factors; it leaves their type and measure undecided. In the 18th century, Newton’s impressed forces are *not* the only possible causes of dynamical change. Such causes could well be Leibnizian ‘live force’ or Cartesian-Malebranchist ‘force of motion,’ as opponents argued in the *vis viva* debates. Kant’s law of inertia declares that only external causes change a body’s state. But the question arises: what is the measure of these changes—how much mechanical effect does an external cause bring about? In answer, Newton offers his Second Law, which Kant lacks. Instead, advocates of equivalence argue, Kant bypasses the need for *Lex Secunda* by having his Reaction Principle govern *directly* changes in motion rather than two forces.¹³ Ergo, they conclude, Kant’s K2 and K3 are equivalent to Newton’s three laws.

This line of argument rests on a key assumption: that Kant’s and Newton’s third laws have the same content and range.¹⁴ But I submit that it requires a more sustained defense if it is to be true. Despite *verbal* similarities between the two men’s claims about action and reaction, reasons for caution abound: unlike Kant, Newton gives precise, technical senses to ‘action’ and ‘reaction’; Kant’s

⁴ “Kant works Newton’s principles into his synthetic principles”—Cohen (1885), p. 245. For V. Mudroch it is a “fact, that the laws of *MAN* are largely identical with those of Newton’s *Principia*.”—Mudroch (1887), p. 78. More recently, P. Guyer has Kant “derive the three laws of Newtonian mechanics by applying the three principles of judgment,” or Analogies of Experience; see Guyer (2006a, 2006b), p. 162.

⁵ “The pure reason described by Kant could last no longer than the Newtonian physics, which was its proper function to justify.”—Gilson (1999), p. 184. Cf. also van Fraassen (2002), p. 8; Guyer (2006b), p. 2.

⁶ Presumably, it means a theory of interactions at sub-luminal speeds through forces that are functions of mass; and it assumes that length and time-interval are well-defined independent of bodies’ states of motion.

⁷ My argument here owes much to Eric Watkins, whose tireless efforts to highlight the post-Leibnizian context of Kant’s mechanics have greatly influenced me. See Watkins (1997) and Watkins (1998a). Here, I build on Watkins’ work, and extend it all the way to Leibniz.

⁸ Kant offers them in “Metaphysical Foundations of Mechanics,” Chap. III of his 1786 tract. He derives them from the Analogies of Experience and an analysis of the concept of matter explicated as “the movable insofar as it has moving force.”

⁹ “Every body perseveres in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by forces impressed”—Newton (1999), p. 416.

¹⁰ “A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed.”—Newton, *Principia*, 416.

¹¹ “In all changes of corporeal nature the quantity of matter as a whole stays the same, neither increased nor diminished.”—Kant (1903 [1786]), p. 541. I follow convention and refer to volume and page numbers in the Prussian Academy edition of Kant’s *Gesammelte Schriften*. Thus, 4: 543 refers to that edition of Kant’s *Metaphysische Anfangsgründe der Naturwissenschaft*. Next two quotes: *ibidem*, 4: 543, 544.

¹² Friedman (1992), p. 145. Credit where credit is due: to my knowledge, among proponents of ‘Newtonian’ readings of Kant only Friedman has made sustained, repeated efforts to account for the discrepancies between Kant’s and Newton’s laws of motion.

¹³ “Since Kant takes the equality of action and reaction, in the first instance, to govern changes of momentum (rather than the forces which produce such changes), he actually does not need to formulate Newton’s second law separately.”—Friedman and De Pierris (2010), fn. 31.

¹⁴ Some 19th-century neo-Kantians first made this move: “As the third law [of mechanics], both Kant and Newton posit the principle of action and reaction.”—Stadler (1883), p. 187.

Reaction Principle has some un-Newtonian traits that call for explanation; Leibniz and his followers had views of action and reaction at odds with Newton's; the foundations of 18th-century mechanics were in flux, and far from uniformly 'Newtonian'; and classical dynamics itself is home to "various statements, mostly quite vague, passing under the title 'principle of action and reaction.'"¹⁵ Together, these factors demand a stricter account of the claimed equivalence between Kant and Newton. Next, I detail how Newton understood his Third Law, and note three ways in which it diverges from Kant's Reaction Principle.

1.2. Action and reaction in Newton's *Principia*

Newton appears to use 'action' and 'reaction' in two ways, but only one in the *official* sense, operative in his mechanics. According to it, 'action' and 'reaction' are exerted on *different bodies*: they are "the actions of two bodies upon each other." And the official *Lex Tertia* requires *Lex Secunda* to give it sense and size: from Definition 4, we learn that "impressed force is the action exerted on a body to change its state."¹⁶ So 'actions' and 'reactions' *just are* impressed forces, whose measure, by the Second Law, is the Newtonian 'change in motion,' or deflection from inertial path in a given time. Ergo, action and reaction cause 'changes in motion' in the different bodies on which they act. These changes must be "reckoned in the direction of their forces," as Newton explains. And the direction of the forces is on the straight line between the interacting particles; if exerted on *bodies*, actions are along the 'line of centers'—the straight line between the bodies' *centers of mass*. The official sense of Newton's Third Law has some defining features. (i) Action and reaction are *identical* with impressed forces; and both are dynamically homogeneous, i.e. forces of the same kind. (ii) Hence, the Third Law *requires* Newton's Second Law for its meaning and measure. (iii) The law holds *equally* for impact, pressures, and distant interactions; it applies directly to both impulsive and continuous, accelerative forces.

When seen against this backdrop, Kant's Reaction Principle baffles. First, it is the second of just two laws of motion, not three—as Newton has, Kant lacks Newton's key statement, the *Lex Secunda*. One may retort that, in *MAN*, the categories of relation fix the number and content of his "mechanical laws." But that is insufficient: Kant asserts *a priori* laws of motion before transcendental idealism—and when he does, he offers two, not three. His 1758 *New Doctrine of Motion* has a Reaction Principle: it is one of two laws of motion, and is very similar to its version in *MAN*; but in 1758 Kant was free from any architectonic constraints imposed by the First Critique. So, the question stands. Second, Kant takes his Reaction Principle to apply primarily to *impact*, or collisions. 'Communication of motion,' to which he restricts the proof of his third law, was a term of art for velocity exchanges in impact.¹⁷ It is another feature that survives from his pre-Critical years, where he applied his Reaction Principle to collisions. Newton has no such strictures on his Third Law, though. In fact, impact hardly appears at all in the *Principia*; the chief and most spectacular application of *Lex Tertia* is to mutual attractions through empty space. But the most puzzling

aspect is the absence of 'impressed force' from Kant's Reaction Principle. The central concept of the *Principia* is impressed force. Its eminence extends to the Third Law: action and reaction *just are* impressed forces. Not so for Kant: his 'mechanical actions' are *effects* of 'moving forces' [*bewegende Kräfte*], a distinct concept. If he meant to ground Newton's theory, his omission of impressed force needs explanation.

So the claim that Kant aims to derive Newton's laws—the gist of the Marburg reading and a vital premise for the Friedman thesis—is implausible without good answers to three questions. Why does Kant have only two laws of motion, not three? Why does he privilege inelastic *impact*? And why does he not make *impressed* force the basis of his mechanics? The reading I offer below has historically sound answers to these questions. But it also concludes that Kant meant his two laws to repair a tradition of metaphysical dynamics deeply influenced by Leibniz. It shows that Kant did not intend to *prove* Newton's laws. Then the question becomes, at most, whether Kant has the conceptual *resources* to support a mechanics like Newton's. I address that issue, briefly, in the last part of this paper; for now, I must make good on my promise. I start by noting that Leibniz had a physics too, not just a metaphysics, and that Kant debuts as a natural philosopher in Germany, a scene deeply shaped by the heritage of Leibniz.

2. Leibniz on action and reaction

For almost a decade in the 1670s, Leibniz believed that ground-level physics had to be an account of particle collision. In his attempt at one, the "theory of abstract motion," he used only velocities and their vector addition to predict the outcomes of impact.¹⁸ Around 1678, he renounces his kinematic past, and offers as fundamental theory 'dynamics,' with 'force' as the central term. Prodded by friends and disciples eager to hear more, in 1695 Leibniz produces a *Specimen of Dynamics*.¹⁹ He says that 'dynamics' straddles metaphysics and the empirical science of motion, and sketches a taxonomy of 'force,' his new genus. There are primitive forces, powers "inherent in all corporeal substance as such," and derivative ones, which he sees as limitations, modifications, or accidents of the former.²⁰ In turn, both types come in two kinds, active and passive. Primitive active force is a true substance's tendency, or 'striving,' to pass from one state to another; derivative active force is in 'phenomena,' as an endogenous attribute of material bodies. It is itself either 'dead'—a mere "urge to motion"—or 'live' [*vis viva*], in bodies actually moved. Primitive passive force underlies that "Resistance, whereby a body withstands not just penetration but also motion";²¹ it grounds impenetrability and "natural inertia" in bodies. Because of natural inertia, he explains, a stationary body remains at rest, and objects get slowed down when they try to move it.

Armed with his new concept of force, Leibniz proclaims to have something better than the kinematic "laws of extension" of his youth; now he can offer "systematic laws of motion." One is "that every action also has a reaction."²² No details come forth in the published part of the *Specimen*. But he takes up the topic again, in the

¹⁵ Noll (1974), p. 32, fn. 3. See also Hamel (1909), pp. 353ff.

¹⁶ Newton (1999), p. 405; my emphasis.

¹⁷ See, e.g., Leibniz's letters to Bernoulli, in Bousquet (1745), letters XIX–XXI; Euler (1738), pp. 159–168; Bernoulli (1742), pp. 7–106; Boscovich (1763), p. 22; Maupertuis (1768), pp. 29–42.

¹⁸ Leibnizian dynamics, and its embedding in his metaphysics of substance, has had several expositors, but few as dedicated and deep as Dan Garber, on whose scholarship I rely here. See, e.g., chap 3–6 of his Garber (2009). For Leibniz's early kinematic physics, see Woolhouse (2000), 143–157. When I quote Leibniz, I use the system of abbreviations common in the field. 'A' is Leibniz (1923), viz. the *Akademie* edition of Leibniz's works, followed by series, volume, and page number; 'E' denotes Erdmann (1840); 'GM' and 'GP' are Gerhardt's edition of Leibniz's *mathematische* and *philosophische* writings, i.e. Gerhardt (1849, 1875), followed by volume and page; 'L' is L. Loemker and trans. (1969); 'WF' is Francks, Woolhouse, and trans. (1998).

¹⁹ Leibniz wrote this essay in two parts, but only the first appeared—as *Specimen Dynamicum*, in the April 1695 issue of the Leipzig *Acta Eruditorum*, 145–157. A complete English version is in WF, 153–179.

²⁰ The exact relation between primitive and derivative forces in Leibniz is notoriously difficult. I have found Paul Lodge's account enlightening; see his Lodge (2001).

²¹ Leibniz, "[On Body and Force]," in GP IV, 395.

²² "...omnis actio sit cum reactione," WF 162; next quotation: *ibidem*, 173 (italics in the text).

unpublished Part II, and adds emphatically that “*there is no action of bodies without a reaction, and that they are equal and in opposite direction.*” This is direct evidence that Leibniz had his own Reaction Principle. Less clear is what action and reaction might be, for his mechanics, unlike Newton’s, is not built on impressed forces. Neither does he admit action-at-a-distance, and so another question is what the range of his principle is.

In writings available to contemporaries, Leibniz’s remarks on action and reaction are scant, often *sotto voce*; and they suggest the following model. ‘Action’ and ‘reaction’ seem to apply to impact in general, not just elastic collisions. An action is the effect of a “force of acting” exerted by an “active” body upon a “resisting” one; its measure appears to be the amount of “new force” acquired by the resisting body as a result. A reaction is the effect of a “force of resistance” put up by the resistant against the active; numerically, it seems to be equal to the “force” lost or spent by the active body so as to overcome the resistance of the former. Let us call it the ‘2-body model’: action and reaction are exerted by (and upon) different bodies. It is the view suggested in *Specimen I*, where the claim that every action has a reaction comes with the gloss, “no new force is produced without reducing an earlier one.” It also emerges in a letter to *Journal des Savans*, as he offers that, over and above extension, the essence of body consists in action and force; and concludes, “*everything that acts must suffer some reaction*, hence a body at rest will not be carried away by another in motion without changing some of the speed and direction of the moving body.”²³ The incoming body, he explains in a follow-up, slows down because the quiescent opposes its advance and effort to drag it along. And the ground of this resistance is “the *natural inertia of bodies*, or that whereby matter resists motion, indeed whereby a body already moving could not carry with it another one at rest without being slowed down.” This inertia of a body at rest requires an incoming body to “employ some force so as to set it in motion.” He restates the point in *Nouveaux Essais*: “natural inertia” makes matter “resistant to motion, so that force must be expended to move a body.”²⁴ It is a “passive force of resistance,” as he calls it in *De Ipsa Natura*; and it resists the “motive force inherent in bodies” already in motion.

In this account, action and reaction stem from *different kinds of powers*. In public claims, Leibniz suggests that his action–reaction duality overlaps with the distinction active vs. passive force. An active force exerts the action; but reaction is grounded in a ‘force to resist,’ hence passive. To confound matters further, sometimes he also assigns different *dynamic roles* to the two bodies in a basic interaction. One is said to be the “agent,” or *active* body, whereas the other is the “reagent,” the body that *reacts* through its “force of reacting.” This transpires clearly in a letter to Wolff, who had asked him to explain how primitive force is modified, e.g. as an object falls freely. Leibniz responds that the best explanation is “by expounding how derivative force changes in phenomena,” i.e. as extended bodies move.²⁵ In turn, a body’s derivative force—*vis viva*, presumably—changes only through impact.²⁶ And he adds:

“Further, you should know that every new derivative force is produced by the intervention of a Reaction. But this Reaction is in

itself indefinite, and depends on some other, agent body [*ab alio agente*] whose direction—as you certainly recall—is opposite to the reagent’s direction. Hence, again, in the struck body there arises compression and, again, the exercise of Elastic force, i.e. the restitution of the Compressed body. [...] Moreover, Reaction presupposes antitypy, the fount of all resistance.”

Little was really clear, in this passage, about the mechanism of action and reaction. Obscure and ambiguous, Leibniz’s account of action and reaction left his disciples in some confusion. Still, a few things were clear. First, they learned from him that the Reaction Principle applies only to action-by-contact, chiefly *collisions*. Second, they also noted his point that the Principle proves or justifies *other laws* of motion. Third, they kept his thought that action and reaction were *dynamically heterogeneous*: they stem from distinct *kinds* of forces. For Leibniz, action results from active force, whereas reaction comes from resistance or ‘passion,’ whose ground is the ‘force of inertia,’ a passive force in his dynamics. It was his distinct move to claim that reaction—but *not* action—is grounded in passivity, hence in the ‘natural inertia’ of matter.²⁷ This duality leads him to suggest another one between active and passive *bodies*, or ‘agents’ and ‘reagents’; his disciples who picked up on it thought they had his blessing to do so. But one aspect of his Reaction Principle always eluded them. As his dynamics lacks impressed forces, they could never see clearly what the *measure* of Leibnizian ‘action’ and ‘reaction’ was.

Thus, two circumstances conspired to endow the post-Leibnizians’ Reaction Principle with a life of its own, quite distinct from Newton’s *Lex Tertia*. For one, they stubbornly ignored the intimate connection between Newton’s Second Law and Third Law. Leibniz and his followers had their own views on force and its measure, and their ideas differed *toto coelo* from Newton’s; hence ‘action’ and ‘reaction’ had to be given fresh meanings. Also, the insight that action (in some sense) must be equal to reaction (in some sense) was already at home in Leibniz’s dynamics, although there it mostly lay dormant. So, for decades before Kant, the post-Leibnizian milieu was the rich soil in which a Reaction Principle grew, at odds with Newton’s in both spirit and detail. As I will show presently, on this soil Kant seeks to clear some ground for his own *Reaktionsprinzip*.

2.1. After Leibniz: Paduan interlude

Though he lost to Newton the race for celestial mechanics, the old Hannover courtier was good at building *un empire leibnizien*, a network of clients and disciples to advance the twin causes of his calculus and dynamics. He saw a chance to reach out to Italy when the mathematics chair at Padua, once held by Galileo, became vacant in 1703. Leibniz lobbied hard to have it given to Jakob Hermann, a young Swiss who had taken his side against Niewentijt.²⁸ A protégé of Jakob Bernoulli, Hermann was left without a patron after Jakob’s untimely death in 1705, and Leibniz was happy to offer his protection. At Padua there was keen interest in hydraulics, so Hermann set out to write a tract on fluid dynamics; at work on it, he resolved to preface it with a book on the “forces and motions of solid bodies.” Thus came to be his *Phoronomia*, the main treatise in particle dynamics on the Continent before Euler’s *Mechanica*.²⁹

²³ Leibniz, “Lettre sur la question, si l’essence du corps consiste dans l’étendue,” (*Journal des Savans*, 18 June 1691), E 113. Next quotation: “Extrait d’une lettre pour soutenir ce qu’il y a de lui dans le *Journal des Savans* du 18 juin 1691,” (*Journal des Savans*, 5 January 1693), *ibidem*. All emphases in the original.

²⁴ Leibniz (1996), p. 123. Next two quotations: *Nature itself; or, the inherent force and activity of created things* (1698), §§11–12, WF 216f.

²⁵ Leibniz to Wolff, 9 July 1711, in Gerhardt (1860), Letter LXVII, p. 139.

²⁶ For Leibniz, even acceleration in free fall, for instance, occurs “through a renewed, continual percussion of the falling body” by a shower of particles of “gravific matter,” “as though in every arbitrarily small time interval some globe or globule struck it.”—*ibidem*.

²⁷ As early as 1688 he conflates reaction and passion; for instance, as he jots down avidly reading notes on Newton’s tract—see Bertoloni Meli (1988), p. 489. By the mid-1710s, he comes to assert that, *unlike* action, reaction stems from inertia, the ground of all passivity in his dynamics.

²⁸ On Leibniz and Johann Bernoulli’s maneuvers to secure the Padua chair for Hermann, see Robinet (1991). On Hermann’s life and work, see Nagel (2005); on his activity in Padua, see Mazzone & Roero (1997).

²⁹ J. Hermann, *Phoronomia, sive de Viribus et Motibus Coporum solidorum et fluidorum libri duo* (Amsterdam, 1716). On its importance in 18th-century mechanics, see Roero (2002), pp. 394ff.

In letters to Hermann, Leibniz had sought to initiate his correspondent in dynamics, the idea of force, its measure and conservation. He alludes to his “metaphysical principles” as the “true founts” of his physics, and offers to impart them to Hermann, who avers that they must be “quite remarkable,” and wishes they adorned his work “like some most precious gems.”³⁰ By the time of *Phoronomia*, the immersion in his mentor’s dynamics is complete. The treatise opens with a brief ordering of force into active and passive. Active force is “motive,” for it either urges a body to move or motion results from it; and it is of two kinds, “live” and “dead.”³¹ In contrast, there is just one passive force, “from which neither motion nor tendency to move results, but consists in that *Resistance* whereby it opposes any external force striving to change the bodies’ state of rest or motion. This force of resistance the great Astronomer Kepler called *Force of inertia*.” To show that it exists, Hermann points to impact with a body at rest, just as he saw Leibniz do: the incoming body slows down after collision, “from which it is clear that the resting body... really has some passive force, which the incoming’s force must break and overcome,” and so use up some of its own “force and motion.”

The reader who saw Leibniz connect the passive force of inertia, resistance and reaction might expect Hermann to do the same; he does not disappoint:

“In this force of inertia of matter is grounded the law of Nature whereby *to every action there is an equal and opposite reaction*. For in every action there is a struggle [*luctatio*] between an agent body and a patient one, and without such struggle no action, properly so called, of the agent upon the patient can be conceived; for otherwise there would be no stable foundations for Mechanics, but any effect would follow from any cause.”³²

Hermann claims that his Reaction Principle expresses a “very well known” law of nature, but in his age only its wording was common currency; the meaning, range and measure of ‘action’ and ‘reaction’ were hotly contested by two rising traditions, Newtonian and Leibnizian. He seems to favor the latter; but as *Phoronomia* is not built on impressed forces, some early readers wondered what he meant by an action met with an equal and opposite resistance. To dispel their perplexity, Hermann adds some clarifications in an appendix he wrote after the treatise went to press. He warns against *equivocating ‘force’ and ‘action,’* and explains that action is really just the application of some force upon a body “which resists, withstands, reacts.”³³ To overcome the patient’s resistance, an agent body must use force:

“Hence in all corporeal action there is a clash between an agent force and the resistance of the patient body, an application of the agent’s force onto the body receiving the action; that is, action itself is equal and contrary to the resistance of the

patient, which is its reaction, because this resistance—or this force of inertia—by the patient body must be removed, so that the patient might be set in motion by the agent.”

The agent uses up only part of its ‘total force’ in this clash; with the remaining force, it moves the patient now devoid of resistance. The Reaction Principle, as Hermann sees it, says that “*in all corporeal action, as much of the agent’s forces is lost as it is gained by the body receiving the action.*” That is his last word on it, as if italics could clarify meanings.

Bafflingly, Hermann’s Reaction Principle plays no active part in *Phoronomia*; though stated early, it is never put to any work. His treatment of central-force motion is confined to 1-body dynamics, so he cannot use the Principle to infer to any mutual attractions, as Newton did. And impact—another obvious candidate—he solves by conservation laws (of *vis viva* and momentum), not his Reaction Principle.³⁴ The equality of action and reaction seems to be a mere decorative appendage, invested with foundational credentials but denied an active role in building theory or solving problems. None of this, though, stopped the Leibnizians from embracing it with open arms.

2.2. *Iter germanicum: the principle returns from Italy*

When *Phoronomia* came out, Leibniz read it more than *oculo fugitivo*. In private, he grumbled that Hermann had been unduly generous to ‘the British.’ If we look closely, we see that he deplored only Hermann’s penchant for geometric proofs—then all the rage in Newton’s Britain—over his analytic calculus; but Leibniz did not accuse the Swiss of borrowing Newton’s laws to build his mechanics. In a letter to the author, he courteously gives only constructive response; beside some advice, there is explicit endorsement:

“The inertia of matter, of which you speak in § 11, is a wonderful thing, and a topic for the deepest research; few have grasped it so far. Amazing consequences follow from it. For if, in matter, we considered nothing beside extension and antitypy, there is no reason why matter [at rest] in a place would resist to [another one] in motion, i.e. would tend to remain at rest. Hence there is no reason why there would be a struggle between agent and patient, since matter at rest is indifferent [to motion], and the least motion would prevail over rest.”³⁵

He praises Hermann’s adopting *his* view of inertia as force of ‘resistance’ and part of the essence of matter, and also of interaction as clash between agent and patient. Predictably, he also supports the conclusion that the patient body, unable to act, merely reacts by its force of inertia. This comes out in his review of *Phoronomia* for the Leipzig *Acta Eruditorum*: “In the introduction, Hermann notes that the rule ‘to every action there is an equal reaction’ follows from the *inertia of matter*, first discovered by Kepler.”³⁶

³⁰ Leibniz to Hermann, June 1712; and Hermann to Leibniz, 4 Aug. 1712, in GM IV, 372, 377.

³¹ Hermann, *Phoronomia*, §§ 7–10. Following Leibniz, Hermann then calls live force simply “Force,” and dead force “Solicitation.” He gives only two examples of solicitation: gravity and centrifugal force. Next two quotations: *ibidem*, §§ 11–12.

³² *Phoronomia*, § 12. The claim that, without reaction being equal to action, no stable laws of motion could be inferred also comes from Leibniz.

³³ Hermann, *Phoronomia*, Appendix § I, 378. Next two quotations: *ibidem*.

³⁴ Hermann’s theory of impact is in *Phoronomia*, Chap. VI “De Regulis motus in collisione Corporum”; conservation of *vis viva* is posited as a hypothesis, § 218; conservation of momentum is derived as a theorem in § 220, but Hermann’s Reaction Principle plays no role in the inference.

³⁵ Leibniz to Hermann, 17 Sept. 1715, GM IV, 398.

³⁶ Cf. *Acta Eruditorum*, no. 1 (January 1716), 2; emphasis in the original. Reviews in the *Acta* were anonymous, but the balance of evidence points to Leibniz as the author of this item. Though letters between Leibniz and Wolff in October 1715 suggest Wolff wrote it, two other indications single out Leibniz. See Ravier (1937), p. 107, no. 250. Ravier does not show his source for the claim, but in a letter of 10 May 1716 to Johann Bernoulli, Wolff writes, “Leibniz composed the review [of *Phoronomia*].” See UB Basel Ms L I a 671, Nr. 12*. I thank Dr Fritz Nagel (Bernoulli Forschungsstelle, Basel) for bringing these sources to my attention and graciously providing me with a copy of Wolff’s letter.

This official seal of approval on Hermann's *Reaktionsprinzip* was not lost on Christian Wolff, a polymath who set out in the 1720s to erect a vast system of rationalism.³⁷ His views are on display in *Cosmologia Generalis*, whose impact on its age is hard to overstate.³⁸ Though he refused Leibniz's idealism about monads, Wolff's epistemology of 'cosmology' is Leibnizian: results are derived either by (loose) deduction or by the Principle of Sufficient Reason (PSR). In his ontology, bodies are extended, fill space, and behave mechanically; all corporeal action is by contact. Leibniz taught him that body has something beyond extension—force, the source of activity—and he retains the insight. Wolffian bodies are composites that act, hence are endowed with "active force." To this, he adds that "in bodies, all changes are by motion," and infers that bodies in motion have "motive force," *vis motrix*.³⁹ This is one half of his notion of body: they "consist of matter and active, or motive force." The second half—matter—is "extension endowed with force of inertia." It is a *vis passiva*, whereby a body opposes "any force that urges it to move."⁴⁰

This *a priori* doctrine makes contact with empirical science through impact dynamics. Wolff takes inelastic collisions to be basic for theory.⁴¹ He distinguishes between "rules of motion" and "laws of motion." The rules are those "according to which motive force is modified in the collision of bodies."⁴² By 'rules,' he really means just *kinematic* equations relating masses and initial speeds to speeds post collision, e.g. Mariotte's or Huygens' rules of impact. But they rest on a deeper basis: "the general principles of the rules of motion are called the laws of motion." To state and prove these deeper laws is the task of philosophers like him:

"These days only a stranger to Mathematics is unaware that in the rules of motion there are general principles, from which these rules can be derived. These principles once established, the rules of motion, i.e. of impact, were proved from them in several ways. Mathematicians assume these laws without proof; but it behooves the Metaphysician to demonstrate them."

These metaphysical principles turn out to be (1) a law of inertia and (2) a law of "the equality of action and reaction." So begins the German rationalist tradition of *two a priori laws* of motion, whose last defender is Kant. It was not uncommon to 'prove' the law of inertia at the time, but Wolff's second law demands our attention.⁴³ Like Hermann and Leibniz, he leaves no room for *impressed* force or Newton's *Lex Secunda* in his foundations; his 'forces' are all Leibnizian. Yet he claims, "the action of one body is equal to the reaction of the other." Then what do 'action' and 'reaction' mean? What is their measure and role in mechanics? And where did he get his Reaction Principle? On this last point, a clue emerges: from the equality of

action and reaction, he says, "we can derive the rules of motion, as Mathematicians know; the illustrious Jakob Hermann in *Phoronomia* and Giovanni Poleni in his book on Castles actually did that."⁴⁴ Apparently, by his second law Wolff means Hermann's Reaction Principle, which he saw Leibniz endorse.

His approach to proving the law confirms this suspicion. He starts with some preliminary distinctions. (1) One is active vs. passive force, as I presented above. (2) Another is that between agent and patient. An agent is the subject of a "change of state, whose reason is contained in the subject which changes that state."⁴⁵ In turn, a patient "suffers": it undergoes "a change of state the reason for which is outside the subject whose state is changed." He illustrates it, unhelpfully, with live entities acting upon dead matter: himself writing a letter or a hypothetical reader squeezing a sponge. But when applied to collisions, the distinction is unstable; unaware of that, he presses on undeterred. (3) Lastly, he separates action from reaction, though obscurely. An action is the causing of a change in state of motion, e.g. from rest to motion or from less to more speed. A reaction is a "resistance," i.e. a causing to lose motion. Wolff is clear that the agent acts, whereas the patient reacts; and that both result in "changes in motion." But he never quite says where these changes in motion are located or how to quantify them. Still, he is firm that action and reaction are *dynamically heterogeneous*: they rest on different *kinds* of forces. An agent body acts through "motive force," the dynamical means for its action.⁴⁶ In contrast, a 'patient' reacts "by its force of inertia." None of these contrasts are new; Leibniz and Hermann have them too, Wolff just spells them out. With these distinctions in place, he goes on to prove his Reaction Principle, in three steps. His conclusion is that, in any collision, the *agent* body spends as much *active* force defeating the *patient* body as the latter has *passive* force to resist. This strongly echoes Hermann's gloss on his Reaction Principle; and it is just as opaque about the quantities involved.

One reason why Wolff's measure of action and reaction remains unclear is his mix of ambiguity and silence about the quantity of his forces, active and passive. To find out what that measure might be, let us look at the actual *range* of his law. Though he puts it forward as a "general principle," it does not really ground all of his mechanics, but holds only for inelastic collisions: "In impact, a body not elastic suffers no other change of motion except that which arises from action and reaction."⁴⁷ And also: "if a body impacting another changes its motion in any way by its force, then it too undergoes the same change of motion in the contrary direction, by the force of the latter." Regrettably, Wolff never explains the direction of these changes in motion—a problem that plagues Kant's model too.⁴⁸

³⁷ Some have warned against seeing in him a mere epigone; still, in natural philosophy Wolff followed Leibniz faithfully. He fought on his mentor's side in the *vis viva* debate; and his mechanics comes from Leibniz and Hermann, not Newton. The best comprehensive account of his philosophy remains Campo (1939). On differences between Wolff and Leibniz, see Corr (1975), Rutherford (2004). On the Leibnizian aspects of Wolff's dynamics, see Campo (1939), vol. 1, pp. 207–55; Ecole (1963). Wolff first defended *vis viva* as the measure of 'force' in "Principia Dynamica," in Wolff (1728); Hermann had four papers in that volume.

³⁸ Wolff (1731); multiple subsequent editions.

³⁹ Wolff (1731), §§ 168–70, 138–41. Next quotation: *ibidem*, §§ 141–2.

⁴⁰ He shows twice that it exists: first, bodies "arise" from physical monads, and they have a passive force; second, he takes from Leibniz the argument that a resting body resists incoming ones, so it must have a force of resistance, and "in bodies, the principle of resistance to motion is called *Force of inertia*, or *passive Force*." See Wolff (1731), § 130.

⁴¹ That is to say, it requires the least number of forces and laws to handle, in his theory. Unlike in Leibniz, elastic impact is derivative for Wolff, because it requires appeal to an additional dynamical principle, *vis elastica*, in order to explain and quantify. Inelastic impact is basic for Kant too, for just the same reason.

⁴² Wolff (1731), § 302. Next quotation: *ibidem*, § 303f.

⁴³ See, for instance, d'Alembert (1743), § 3f.; Euler (1736), §§ 56–70. Next quotation: Wolff (1731), § 346.

⁴⁴ Wolff (1731), § 527. Poleni was a professor of mathematics at the University of Padua. Recall that Jakob Hermann had also taught at Padua from 1707 to 1713. He sided with Leibniz in the *vis viva* debate; cf. his letter to Conti in Poleni (1729). Wolff's reference is to Poleni's *De castellis per quae derivantur fluviorum aquae*; cf. Poleni (1718). There, he treats elastic collisions in §§ 119–30. But Wolff misleads on this point: neither Hermann nor Poleni use any law of action and reaction to derive their rules of impact.

⁴⁵ Wolff (1730), §§ 714f. Next quote: *ibidem*.

⁴⁶ Wolff (1731), §§ 132–5. Next quote: *ibidem*, §§ 313, 316.

⁴⁷ Wolff (1731), § 382. Next quote: *ibidem*, § 350.

⁴⁸ What he *ought* to say is that bodies act on each other along the straight line between their centers of mass. Wolff's Figure 3—a drawing in *Cosmologia* to illustrate his claim about reaction being contrary to action—is inconclusive, as it depicts a head-on collision. He never really discusses oblique impact, which would have forced him to be explicit about the meaning and measure of 'action' and 'reaction' in his model.

2.3. *Corpus quiescens: the paradigm and its aftermath*

Wolff's Reaction Principle fuses strands from the metaphysical dynamics of Leibniz and Hermann. From them he learned that interaction is a clash of agent and patient, active moving force and passive force of inertia; that action and reaction, though equal, rest on unlike forces applied by asymmetric actors; and that the *Reaktionsprinzip* governs action by contact, chiefly impact. To this, he adds that it is one of two a priori laws of motion, and that it "grounds" the rules of impact, which natural philosophers must show.

A conjunction of factors made Wolff's account prevail in Germany.⁴⁹ As a result, his model of action and reaction becomes the orthodoxy, and disciples rehearse it for decades. They inherit his taxonomy of force: "*Motive force* is that by which a body is able to move from its place. By the *force of inertia*, a body resists motion."⁵⁰ They import his asymmetry of interaction: "If a [resting] body is struck by another in motion, it suffers: and the latter acts on it."⁵¹ They admit that to be a patient is to react—by resisting: "Thus if a thing resists, its resistance is a reaction by the patient to the agent."⁵² Metaphysicians friendly to his dynamics spell out his link—taken from Leibniz—between passivity, reaction and resistance: "The reaction by which the patient steadily diminishes the agent's action... is called *resistance*."⁵³ The Wolffians adopt his grounding of resistance in inertia: "That whereby a body resists motion is called *Force of inertia*, or *passive force*, a term common in Mathematics."⁵⁴ And they trust his claim—adopted from Hermann, approved by Leibniz—that the Reaction Principle follows from *vis inertiae*: "No corporeal action is without a contrary action. A body's contrary action springs from its force of inertia. And by a contrary action, a body resists motion. But it resists motion by the force of inertia."⁵⁵ Like him, they make the Principle into the second of two laws of motion: "The second law of motion is: a body's resistance is always as great as the action of that which collides with it."⁵⁶ And they repeat his point that the laws must be shown "to have their ground in general cosmology." Lastly, they borrow his division between rules and laws, and the restriction to impact: "Rules of motion are those whereby motive force is modified in the collision of bodies. Laws of motion we call the general principles of the rules of motion."⁵⁷

The paradigm for this view of action and reaction was *impact with a body at rest*. Leibniz set out in dynamics studying collisions, like most of his coevals; and he abhorred action-at-a-distance: he deemed it 'barbarism in physics,' an animus his disciples kept. As a youth, he saw rest and motion as dynamically distinguished, later professions of relativity notwithstanding. "All corporeal action is motion," he writes in 1668, stirred by Hobbes.⁵⁸ Soon he infers, as he drafts *Theoria Motus Abstracti*, that "a body at rest does not

act." He means that resting bodies are causally inert: in a vacuum, he predicts, a moving body striking one at rest will drag it along with no loss of motion. As he discovers 'force,' he also sees the error of his youthful way, but his remedy startles: now he avers that rest is causally efficient, but by *another kind* of force than moving bodies are. That 'force of rest' he dubs "natural inertia," and sells it by pointing to impact with a stationary object: "We notice in matter a quality some have called *Natural Inertia* by which a body somehow resists motion, such that force must be employed to move it."⁵⁹ His early view still echoes: though efficacious, a body at rest does not act, but resists; it is passive, not active. So he calls inertia "this passive force of resistance."

Hermann missed none of these clues: he too declares bodies to have a "passive force," called "force of inertia." He follows Leibniz in saying that "it shows itself plainly" in things at rest—as they collide with moving ones. The moving is "the agent," and it loses speed after impact; ergo, the resting has a passive "faculty of resisting" that the agent must "break and overcome." And in this force of inertia "is grounded the law that *to every action there is an equal and contrary reaction*," Hermann infers abruptly, to Leibniz's delight.⁶⁰ Both agree that impact with a body at rest discloses the equality of action and reaction, as they understood it. Then Wolff follows suit: "every body resists motion," he says, or else a moving body that struck one at rest would push it along, its speed unchanged. Ergo, in body there is a "principle of resistance"; because of it, "when body B moves another body A at rest or changes its motion as it moves: B acts, whereas A suffers."⁶¹ This duality of roles in impact then leads Wolff to prove that the patient resists just as much as the agent acts—his idea of equal action and reaction. Once they illustrate it with stationary bodies, Leibniz and Wolff extend the passive force of inertia to the 'weaker' body in a collision. But the extension is problematic, so they do not dwell on it.⁶² Impact with a resting body was the darling of the Leibniz-Hermann-Wolff tradition—and the source of its downfall and rebirth, at the skillful hands of Kant.

3. Kant revises the Leibnizian consensus

At mid-century, the Hermann-Wolff model of action and reaction held sway over German rationalist dynamics. But it was open to censure from two sides. One is external: it takes Newton's side and wrecks the model with just two blows.⁶³ Another is criticism from within, as it were: it finds internal tension and relieves it by tweaking the model, instead of replacing it altogether. Kant takes the latter route, in the 1758 *New Doctrine of Motion and Rest* (NL).⁶⁴ To save the model, he casts out the force of inertia, ergo resistance and passivity; but this lets him keep much of the rest, it turns out. I have reconstructed the argumentative structure of NL

⁴⁹ Thomas Ahnert has documented the 'hesitant and slow' reception of Newton in Germany. Cf. his (2004).

⁵⁰ Winckler (1735), § 624.

⁵¹ Gottsched (1735), § 359.

⁵² Hanov (1762), § 4.

⁵³ Burkhäuser (1771), § 624.

⁵⁴ Thümmig (1735), vol. I, chap. II, § 31.

⁵⁵ Winckler (1735), §§ 684–5.

⁵⁶ Gottsched (1735), §§ 366, 370.

⁵⁷ Stiebritz (1744), §§ 302–3.

⁵⁸ Leibniz, *Confessio Naturae contra Atheistas*, GP IV, 110. Next quote: Leibniz, "Vorarbeiten zu *Theoria Motus Abstracti*, 1669–70," Theorem IV, A 6.2, 169.

⁵⁹ Leibniz, *Lettre sur la question, si l'essence du corps consiste dans l'étendue* (1691), G IV 464. Next quote: Leibniz, "Nature Itself," § 11, WF 216.

⁶⁰ Hermann (1716), §§ 11–12.

⁶¹ Wolff (1731), §§ 129, 133; emphasis added.

⁶² The 'weaker' body also moves, so it must have 'moving force' besides a 'force of inertia'; then it is mysterious why, in impact, it should exert the latter rather than the former. Second, a moving body is only relatively passive: with respect to the 'stronger.' A resting body is passive *absolutely*—it has no active force.

⁶³ Both charge that it misunderstands the nature of inertia, action and reaction, whose correct account is in Newton's *Principia*. One objection stresses that action and reaction are impressed forces, not effects of active and passive force, respectively. The other points out that both bodies in an interaction have a force of inertia, not just the 'patient.'

⁶⁴ Kant (1912 [1758]), 13–26; hereafter 'NL'. References to NL are by page and line number in this edition. An English translation with notes by Eric Watkins is forthcoming from Cambridge University Press.

elsewhere; here, I focus on his early view of action and reaction and its post-Leibnizian origin.⁶⁵

Kant seeks to expel *vis inertiae* as incoherent and “excogitated without need.” The charge of incoherence is astute, and it cuts deep. Plainly, a body at rest is balanced on all sides, in “equilibrium of force,” as he puts it.⁶⁶ Yet champions of *vis inertiae* as a ‘force of rest’ mean it as an *unbalanced* striving toward the ‘agent’: “The force of inertia is a striving exerting itself against the striving of the active body,” Wolff had said. Kant pounces:

“But how is it that, as soon as the impacting body touches the one at rest, the latter is supposed to suddenly change itself [from equilibrium] into a state of motion or striving directed at the approaching body, in order to destroy in the latter a part of its force?”⁶⁷

To show it is needless, again he singles out *vis inertiae* as the *force in bodies at rest*—as it was for the Leibnizians—and argues that *in impact there is no true rest*: “what one falsely took for rest with regard to the impacting body is in fact a motion relative to it.”⁶⁸ And *this* he shows by refuting the “common concept” of motion and rest, and replacing it with his own. He claims the common view sees motion as “absolute.” It has two versions, which he refutes, then infers, “I should never use ‘motion’ in an absolute sense, but always relatively.” His point is that motion is *relational*: we must not judge that a body just moves, but that it moves *with respect* to other bodies—for motion really is a relation between body and body (as is rest). Still, motion is *not* relative, i.e. *arbitrary*: for every body, a privileged relation exists as its *true* ‘respective’ motion; it is the relation to another body with which it interacts. But the relation is “mutual,” i.e. *symmetric*: if A moves respective B, then so does B respective A. Ergo, in every impact, both move respective each other. Further, it is a relation of “mutual approach,” and each has a share [*Anteil*] in it. As we have “not the least cause” to assign a greater share to one over the other, we must give each body an equal share.⁶⁹ So their motions ‘respective each other’ are equal and opposite. Quantitatively, his doctrine translates as follows. When A and B collide, their “mutual relation” is their *relative speed*; to compute their shares, he takes that speed and allots it to each *inversely as their masses*. Their speeds are now unequal, but they have equal *scalar momentum*. These are their ‘motions respective each other’—as seen in a privileged ‘space,’ in which their CM rests.

From his analysis, Kant draws two “corollaries.” One is evident: in a 2-body collision, each moves respective the other, neither is at absolute rest.⁷⁰ Another says: “In impact, action and reaction are equal.” It is his Reaction Principle, which he states without proof. But we can show it to follow from his argument. Recall that by his new doctrine of motion, (1) any two colliding bodies move *equally* respective each other. Now add two premises: (2) any moving body has “moving force,” and (3) a body’s action is proportional to its moving force. Hence, by (1) and (2) any two bodies “collide with equal force,” as Kant says. Ergo, by (3) their actions upon each other are equal. He does not state (2) and (3) explicitly because his

opponents accept them too. Wolff had posited that “*a moving body is endowed with active force*,” or “*moving force*, as it adheres to local motion”—and disciples agreed.⁷¹ Kant’s real departure is his theory of motion, but once it is granted, he derives a *Reaktionsprinzip* a priori just as they did, though by another route.

The last part of *NL* reveals that his corollaries are in fact laws of motion as the Wolffians meant the term. He uses them in an “explication of the laws of impact by the new concept of motion and rest.”⁷² But these ‘laws’ are just kinematic formulas for predicting the speeds of two bodies post collision, not genuine dynamical laws; he calls ‘laws of impact’ what the Wolffians had called ‘rules of motion.’ His first example is *inelastic impact with a body at rest*—a telling choice. He explains that, by his Corollary I, the stationary body just *appears* to rest; in fact it moves ‘respective’ the incoming. Next he works out their respective motions, so his account switches to the collision’s CM-frame. In it, the bodies collide with equal ‘force.’ By Corollary II, their actions on each other are equal; so, he infers, they come to relative rest—the *real* outcome of the collision. The *apparent* result, though, is what an observer sees in the ‘space’ where one body was initially at rest: post collision, both bodies move together with the residual speed of the incoming. As a philosopher, his task is to find the principles—the *dynamical* laws, such as his Reaction Principle—of the real nature of corporeal interaction, which then can explain the *kinematic* regularities of their apparent motion in impact. This is the gist of a passage in *NL* strongly redolent of the Wolffian program of grounding the rules of impact in a priori, philosophical laws of motion:

“Yet by the name ‘laws of impact’ we understand not just the rules of the relation that colliding bodies enter into (respective each other) but also, primarily, the change of their outer state with regard to the space in which they exist. But this is, properly speaking, just the outer phenomenon of what occurred immediately between them; and it is the latter that we need to know.”

There is good evidence that Kant’s real target is the Hermann-Wolff consensus. Some is circumstantial: as he attacks the ‘force of inertia’ he calls it a *Bestrebung*, his German for their Latin *nisus*; he uses *leidend* to render *patiens*, their term for the resting body that ‘suffers’ by its force of inertia; and claims to prove the equality of action and reaction “without a need to invent a special kind of natural force,” i.e. *vis inertiae*.⁷³ To read these as clues that he argues against Newton would make Kant look confused: the Briton never claims that *his* force of inertia is shown in bodies at rest, that it turns them into ‘patients’ or that it proves *his* Third Law.⁷⁴ But the Wolffians did. Plus, Newton was *not* alone in construing true motion as absolute motion, i.e. translation in absolute space. Some Leibnizians did too—notably Hermann, whom Kant had read: “We say bodies *move* absolutely when they constantly change contact with the parts of infinite immovable space.”⁷⁵ But the key support for my reading is the strong similarity between Kant’s and the Wolffians’ Reaction Principles. Like theirs, it is his second of two laws of motion; both he and

⁶⁵ See Stan (2009).

⁶⁶ Kant, *NL* 2:20. By ‘equilibrium of force,’ Kant means that if outer forces act on the body, they must be equal and opposite; and any inner urge to move in one direction is countered by an equal urge in the contrary direction. Next quote: Wolff (1731), § 319.

⁶⁷ Kant, *NL* 2:20f.

⁶⁸ Kant, *NL* 2:20.

⁶⁹ Kant, *NL* 2:18.

⁷⁰ “Every body with respect to which another one moves, is also in motion relative to the latter; and it is impossible for a body to collide with another one absolutely at rest.”—*NL*, 2:19. Next quote: *ibidem*.

⁷¹ Wolff (1731), §§ 135, 137 (original emphasis); see also Stiebritz (1744), Part III, Sect. II, § 137; Thümmig (1735), Part II, §§ 35–6.

⁷² Kant, *NL* 2:23–5. Next quote: *ibidem*, 2:24.

⁷³ Kant, *NL* 2:20.

⁷⁴ Daniel Warren likewise seems to think the early Kant proposes here a doctrine of force that “does not have as its model a Newtonian idea of attractive force.” Cf. Warren (2001), 111 fn 7.

⁷⁵ Hermann (1716), § 1; Hermann distinguishes apparent from true rest in *De Mensura Virium*, from Hermann (1728), § 2. Kant refers to Hermann’s *Mensura Virium* in his *Estimation of Living Forces*, Ak. 1:43.

they derive it a priori; neither takes it to denote *impressed* forces; they all restrict it to collisions; all use it to derive the rules of (inelastic) impact. Seen in this light, Kant is their internal critic, not external enemy: he does cast off their *vis inertiae*—and so the agent-patient duality, the dynamical heterogeneity of action and reaction—but keeps all else in their model. Most significantly, he illustrates the equality of action and reaction with the same paradigmatic case: inelastic impact with a body at rest.

Alas, he inherits their vagueness about the measure of action, reaction, and their link to the ‘moving force’ of bodies in impact. Is Kantian ‘action’ (1) the *same* as his ‘moving force,’ (2) the *effect* of this force, or (3) the force *spent* by a body in colliding with another? How does one measure action and reaction? Are they momentum increments mdv —infinitesimal degrees of ‘motion’ transferred by one body to another in an instant? Or are they finite changes in momentum, i.e. Δmv ? These are not pedantic niceties, but crucial issues for deciding the range of his Reaction Principle, including whether it can support a mechanics like Newton’s. His views on force, action and reaction in 1758 make it very hard to answer conclusively the issue of their measure. In part, it is due to his chosen paradigm: inelastic *frontal* impact *relative to CM*. There, the bodies’ initial scalar momenta are equal, and equal to the changes in it; and the direction of their moving forces coincides with their ‘line of centers.’ Such equalities and coincidences help Kant obtain results a priori; but they obscure key details of his foundations for mechanics. The *historical* reason for this obscurity is that he takes over a view that was itself ambiguous about the locus and measure of action and reaction: the Leibniz-Hermann-Wolff *one-dimensional* model of impact as a clash of forces. Kant fixes it, but his remedy has limits. Let us now see how his Reaction Principle evolves.

3.1. Kant’s ‘third mechanical law’

In 1758, Kant had restricted his a priori law to inelastic collisions; in 1763, he appears to extend the range of ‘mutual action’ to cover impact, pressure, and attractions (presumably at-a-distance).⁷⁶ The 1780s see a watershed in his philosophy of physics: concepts and principles of the pure understanding are now its official epistemic base; and his table of categories is a blueprint for the disciplinary architectonic of “pure natural science.” The result is a geometric kinematics of force-free, unconstrained particle motion (Phoronomy), a theory of matter (Dynamics), three laws for interactions (Mechanics), and a theory of true motion (Phenomenology).⁷⁷

Kant takes mechanics to be the science of “communication of motion,” or *apparent* transfers of velocity between extended bodies with mass. Philosophy grounds mechanics in two ways. One is by securing apodictic certainty for some of its laws, viz. K1–K3 of Section 1.1. Another is by explaining the possibility of communicating motion in mechanical processes. This requires that matter be thought of as “the movable insofar as it, as such, has moving force.”⁷⁸ That is to say, mechanics presupposes a body to be (finite)

extended, movable matter endowed with ‘moving force’ simply by being in motion, *if* it moves (and he proves that all bodies do move). It is worth noting that ‘moving force’ is the *only* force in his mechanics; he makes no mention of Newton’s duality *vis insita* vs. *vis impressa*. Whatever merits the claim has that Kant grounds ‘Newtonian’ mechanics, plainly his is not built on impressed forces. In Kant, a body’s ‘moving force’ is the *property* causally responsible for changes in the motion of *other* bodies, and is the *sole* mechanical property available to explain them.⁷⁹ ‘Moving force’ is *endogenous* to its body: a ‘haveable’ adhering to it as that whereby the body is “movent [*bewegend*] by inherent motion,” viz. able to move bodies in its path. *Sed contra*, Newton’s impressed force is *not* a property but an *action*, and “consists solely in the action and does not remain in a body after the action has ceased.”⁸⁰ Kant’s and Newton’s mechanics differ both in their taxonomy of force and in their ontology of mechanical agency.

In Kant’s mechanics, the basic event is an encounter between two bodies endowed with ‘moving force.’ His “mechanical laws” ground predictions about its outcome. By his law K1, the system’s mass stays unchanged throughout the event; by K2, neither body can self-accelerate. Ergo, only externally-induced changes in speed are allowed. To find out what they are, he offers the “third mechanical law”: “In all communication of motion, action and reaction are equal to one another.” Here, I am chiefly after what he means by ‘action’ and ‘reaction,’ as he does not have ‘impressed force’ to give them meaning along Newton’s lines. But first, I must examine how Kant obtains his law. He states it, then offers a proof, meant as a genuine derivation, not rhetorical feint.⁸¹ To follow it, we need to grasp two distinctions he makes elsewhere. (1) For mechanics, relations can be *active* or *inert* (the latter is my term, but it fits his thought). The active obtain between bodies causing ‘mechanical’ changes in one another, i.e. changes in *inertial state*. He describes them as “relations of efficient causes related merely to each other,” and calls them “active relations between matters in space.”⁸² A relation that leaves a body’s *mechanical* state unchanged is inert: it lacks mechanical efficacy. Such is “merely phoronomic” motion, or uniform translation, when a body “changes its outer relations to a given space.” In it, one relatum (the space) has “no influence at all on the rest or motion” of the other (the body).⁸³ (2) Kant demarcates absolute from relative space. The latter is a *finite volume*: a space “designated by what can be sensed,” *inside* which we may “view the body as moved”; the volume is “the space of its motion.”⁸⁴ For any relative space, we can *think* another “material space” large enough to count the relative as movable in it; we “abstract from the matter” of the larger, and represent it as a “pure, non-empirical, and absolute” space; and we *assume* it, in thought, to be immovable.⁸⁵

Kant’s proof of his law has two parts, an inference and a construction.⁸⁶ The inference is to prove that action equals reaction, and the construction applies their equality to a special case of ‘communication of motion’—direct impact with a body at rest. He starts from an insight: when motion is communicated, two bodies are in *active* relation: at least one of them causes “external action.” From

⁷⁶ Kant, *Negative Magnitudes* (NM), Ak. 2: 195.

⁷⁷ For an excellent account of the unity of Kant’s argument, see Watkins (1998b).

⁷⁸ MAN 4:536.

⁷⁹ He is careful to add that bodies, being material, also have *dynamical* forces, e.g. repulsion, which they exert in impact. See MAN 4: 536f. Next quote: *ibidem*, 539.

⁸⁰ Newton (1999), 405.

⁸¹ Kant’s faith in his *Beweise* is explicit: in the preface, he is upbeat that his “system,” or metaphysics of body, is “certainly capable” of attaining the full rigor of proofs in mathematical physics—MAN, 4: 478.

⁸² MAN 4: 545. Next quote: *ibidem*, 4: 482.

⁸³ MAN 4: 482. The first are relations *between* matters in space: the relata are bodies, but the relation has a *kinematic* aspect: relative speed, the rate of change of distance between *bodies*. The second are relations *to* a space: purely kinematic relations between *a* body and a *space*.

⁸⁴ MAN, 4:481; 487 (Remark).

⁸⁵ MAN, 4: 481. Clearly, I cannot touch upon all the intricacies of Kant’s absolute space here. As my topic is not his Phenomenology, I trust the brief exposition above suffices to explicate his proof of the Reaction Principle.

⁸⁶ On this point, I am in agreement with Eric Watkins (if I read him correctly), Watkins (1998b), p. 584. Here, I mostly try to spell out steps and assumptions implicit in the proof.

“general metaphysics,” he imports a principle to complete the insight: “all external action in the world is mutual action.”⁸⁷ Hence all active relations must be thought of as mutual. And so is any change in these relations. But “all change of matter is motion.” So, if a body moves respective another, we must think the latter to move too, respective the former. Next, he infers a point he needs for his construction: in a 2-body mechanical event, neither is absolutely at rest. Though one may *appear* to rest relative to the *space* where we observe it, in fact it truly moves—relative the other *body*; and so does the space where it appears stationary.

Kant has just proved that, for any two bodies in active relation, their motion is mutual: “one body approaches every part of the other as much as the latter approaches every part of the former.” Then he argues that each has a share in this mutual motion, and reaches for the Principle of Sufficient Reason to infer that the shares are equal: “for there is no reason to ascribe more motion to one than to another.”⁸⁸ And we may disregard any relation each has to the “empirical space” where we observe them: it is inert, so “nothing here depends on it.” Hence the bodies’ mutual motions are “determinable purely in absolute space.” In a footnote he explains that, for mechanics, the measure of motion is mass times speed.⁸⁹ It follows that in absolute space, where the bodies’ true motions are equal, their speeds are inversely as their masses: in it, the system’s center of mass rests. As their true motions are equal, they have equal ‘moving force,’ the basic trait of matter in his mechanics. But by ‘moving force’ bodies act on each other in the communication of motion. Ergo, their actions are equal: for any action (of a body) there is an equal counter-action, or reaction (by another body mutually acting on the former). Q.E.D.

He adds that the law of equal action and reaction is the “necessary condition” for constructing the communication of motion. By that, he means a procedure to represent the initial conditions and outcome of an interaction by purely geometric means. His third law guides the procedure by teaching (1) how to construct geometrically the true motions of bodies ‘respective each other’ from their apparent motion, and (2) how to construct “in the appearance,” or the observer’s relative space, the kinematic outcome of their mutual action. Thereby the outcome—their speeds post interaction—can be ‘read off’ the geometry of the situation, i.e. *inferred a priori*, without recourse to empirical help. In his proof of the law Kant inserts such a construction, for *frontal inelastic impact with a body at rest*—the same paradigm he chose to illustrate his Reaction Principle in 1758.

With his law proved for direct collision, Kant then claims that impact differs from “traction” [*Zug*] “only as to the direction in which matters resist each other in their motions,” and concludes abruptly, “in all communication of motion action and counteraction are equal.”⁹⁰ But he offers *no proof* for cases beyond frontal impact; nor is it obvious that his *a priori* mechanics has the strength to bear the extension (more on that below). In a Note, he infers a “law of nature not unimportant for general mechanics”: that all bodies are

movable in impact. Of course, it is because they *move actually*, in his sense of ‘respective motion.’ This law is his former Corollary I in *NL*—and a last dig at Wolff, now unable to protest.⁹¹ Confident he can prove his third law in all generality, he offers it to Newton, who “did not trust himself to prove [it] *a priori*, and so had to appeal to experience.”⁹² He scolds those who resorted to a “special force of matter,” *vis inertiae*, to prove that action equals reaction. Knowing the origin of his third law, we know whom he means: the Wolffians. It is 1786, and Kant still fights the last war—against the Leibniz-Hermann-Wolff model. Not just his targets are the same, so is his tactic to defeat them: the parallels with his 1758 proof for the equality of action and reaction are astounding. With his young self, the Critical Kant shares (i) the nature of ‘respective motion’ as initial premise; (ii) the claim that bodies have shares in it, and the inference to equal shares; (iii) the denial of true rest; (iv) the choice of direct impact as paradigm; (v) the use of PSR to prove that respective motions, ergo ‘moving forces,’ are equal; (vi) the view of action and reaction as effected by the ‘moving force’ of bodies in true motion; (vii) hence the attempt to prove his Reaction Principle without *vis impressa* and Newton’s Second Law; (viii) using the Principle to derive rules of inelastic impact: in *MAN*, the communication of motion he constructs is a geometric form of the Wolffian ‘rule of motion’ for impact with a resting body.

Not least, a subtle but deep continuity: in both 1758 and 1786, Kant *starts* with a doctrine of true motion and infers his Reaction Principle *from* it; his concept of true motion *precedes* the law, and is defined *independently* of it. This seems to suggest a need to qualify the influential thesis that Kant “conceives the laws of motion rather as conditions under which alone the concept of true motion has meaning: that is, the true motions are just those that *satisfy* the laws of motion.”⁹³ However, the Critical Kant betrays a terminological indecision, which leads to uncertainty—over the *locus* of action: he uses ‘effect’ or ‘action’ [*Wirkung*], ‘acting’ [*Handlung*], and ‘counteraction’ [*Gegenwirkung*] to denote quantities that are equal, but only thanks to the special nature of the interaction and Kant’s choice of description: the collision is frontal, inelastic, and referred to the CM of the system.⁹⁴ These equalities help his proof at the price of obscuring his concepts.

Now I can answer the three questions I set out with as criteria for a good interpretation of Kant’s third law. (1) Why only two laws of motion? Because Kant first derives *a priori* laws of motion by constructive engagement with a Wolffian account of interaction that was itself based on just two laws. (2) Why inelastic impact as a paradigm? Because the Wolffians’ *a priori* laws were meant to yield the kinematic ‘rules’ of inelastic impact, as do Kant’s. What some have found “weird” or “obscure” is, on my reading, predictable and transparent.⁹⁵ (3) Why no impressed forces in Kant’s mechanics? Because his early view of interaction repairs a preexisting account (the Leibniz-Hermann-Wolff model) that lacked impressed force; instead, it had ‘moving force’ and ‘force of inertia.’ Kant discards the latter, but keeps *vis motrix*. Arresting continuities

⁸⁷ *MAN*, 4: 544f. As has been noted, this principle is *not* Kant’s Third Analogy, but the more robust claim that all causality between “matters in space” is mutual. See Watkins (1998b), p. 574. Next quote: *MAN*, 4:545.

⁸⁸ *MAN*, 4: 545 (Proof). Next two quotes: *ibidem*.

⁸⁹ *MAN*, 4: 547.

⁹⁰ *MAN*, 4: 546. It’s unclear what exactly Kant means by ‘traction.’ He proves his law for *direct* impact; and early in his *Mechanics* he mentions a hypothetical comet dragging the Earth *directly* in its wake (4: 537). But that is much narrower than the general case of two bodies *orbiting* around each other under forces as the inverse powers of their distance. Kant’s *Zug*, in contrast, is closer to Wolff’s *tractio*, whereby he meant an ‘agent’ dragging a ‘patient.’ Cf. Wolff (1731), §§ 339–40. Wolff believed such ‘tractions’ were also subject to his law of action and reaction. But clearly, in such cases, both terms must be given *fresh meanings*. To handle Wolffian ‘tractions,’ we need a work-energy theorem, not Newton’s Third Law. Then ‘reaction’ would measure the work done by the dragger (*trahens*), and ‘action’ the change in the *vis viva* of the dragged (*corpus, quod trahitur*).

⁹¹ In Wolff (1731), pp. 388–9, Wolff had posited that, if a very small body moving very slowly collides with a large resting one, it will be brought to rest without moving the stationary. This is because “the resting body’s force of inertia will prevail over the force of the incoming body.”

⁹² *MAN*, 4: 549 (Remark I). Next quote: *ibidem*.

⁹³ Friedman (1992), p. 143 (emphasis in the text).

⁹⁴ If *Wirkung* denotes a body’s change in momentum *after* impact, *Gegenwirkung* means the body’s momentum *before* impact, and *Handlung* refers to the *other* body’s momentum or perhaps the momentum it loses in collision. Kant’s vocabulary strongly evokes Leibniz’s 1-body model of action and reaction: in Kant’s setup, *Wirkung* and *Gegenwirkung* are equal and opposite, but they are located in the *same* body, *before* and *after* impact.

⁹⁵ P. Kerszberg finds it weird in his (2009), p. 66; R. Palter thinks Kant’s motivation and proof are obscure, in his (1973), p. 105.

between his early and Critical concepts of action and reaction explain the un-Newtonian traits of Kant's third law in *MAN*. So far, I have enlisted history to signal differences between Kant and Newton; below, I sketch some reasons to doubt they are easy to bridge.

3.2. The limits of Kant's a priori mechanics

Kant warns that his proofs are not rigorous, but claims bravely that his laws could be proved in all rigor. Then it is fair to ask what that requires, and if he has the resources for a full proof of his third law.

The law he does prove holds directly for head-on collisions between *particles*. As Kant takes his mechanics to be about the motion of extended *bodies*, the law's range must be limited to bodies in *pure translation*, i.e. whose parts do not rotate around an axis. If the colliding bodies rotate, Kant's law does not suffice to construct their communication of motion. Another principle of mechanics must be added: conservation of angular momentum.

It is quite hard to see how to apply Kant's law beyond his chosen case. Consider the oblique impact of two homogeneous, non-rotating discs with velocities at an angle. To handle it, we must resolve their motions into two components: frontal (along the line of centers) and parallel (perpendicular to that line, and so parallel to each other). When bodies in pure translation collide, only their frontal components change. To find their motions post impact, for each body one must compose its ensuing frontal component with its (unchanged) parallel one. Yet attempts to tame oblique impact strain Kant's theory. His mechanics rests on 'moving forces,' and 'actions' are effects of such forces; but his silence about the *directions* of force and action leads to tension:

(1) His mechanical forces are impetus-like, so their natural direction is the line of the bodies' motion. But we know that bodies act on each other along their line of *centers*. In oblique impact, these two lines differ. To keep his mechanics coherent, Kant ought to say that action (and reaction) is caused not by the whole moving force, but only a part of it: the frontal component. Then it is unclear that he can still prove the equality of action and reaction by his master argument. (2) Or he could say that actions are *only* along the line of centers. Then he needs to explain what action is, and how it relates to 'moving force' if their directions differ. And he must establish (2) a priori. To do so, he has to prove (i) that every body has a unique center of mass, and (ii) that mechanical actions between two bodies are collinear *in recta per centra amborum transeunte*, as Leibniz puts it. These are required premises if Kant is to *prove* a priori a law that stands a chance of being equivalent to Newton's *Lex Tertia*. Kant asserts, in passing, that the 'active relation' of his two colliding bodies "depends only on the line between them." But we cannot know with certainty how he means it: in his example—direct impact—the line of motion and the line of centers coincide; in fact, he seems to favor the line of motion, which pushes him towards (1) above, away from (2).⁹⁶ Neither do we know what a priori argument he might give for (2), unless we let him take Wolff's route and resort to the Principle of Sufficient Reason.

I chose my example strategically, so as to highlight a problem for the 'equivalence thesis' that Kant's third law is fully equivalent to the *Principia*'s Second and Third Laws, hence could support Newton's mechanics. Oblique impact is akin to Kant's original example yet has a feature in common with Newton's unique application of *Lex Tertia* to mutual orbiting in empty space. The feature is that in both cases the bodies' lines of motions *differ* from their line of action. The interpreter's challenge is to explain, on Kant's behalf, in

what *direction* Kantian 'actions,' hence 'changes in motion,' occur in oblique collisions. The interpretation must use only domestic resources—Kant's concept of 'moving force' and his kinematics—and live up to a non-negotiable standard, a priori derivability from his metaphysical *Körperlehre*.

Extending Kant's law beyond direct impact is hard because he lacks just what allows Newton to succeed: impressed force and the Second Law. For Newton, action and reaction cause mutual 'changes in motion'; as both action and reaction are impressed forces, the 'change in motion' is in the direction of the force, by *Lex Secunda*. And impressed forces act along the line through the body's center of mass. The same law measures Newtonian 'changes in motion': they are equal to the body's deflection from its inertial path in a given time. To ground these claims, "Experimental philosophy proceeds only upon Phenomena and deduces general Propositions from them *only by Induction*,"⁹⁷ Newton writes—thus basing *Lex Tertia* on experience, Kant notes with reproach. Advocates of the equivalence claim must show that Kant can get Newton's Third Law without appeal to experience. The obstacles, I hinted above, are daunting, because Kant's thought on action and reaction was shaped by a milieu—the Wolffians—that ignored *Lex Secunda* resolutely. By correcting them from within, Kant appears to inherit some of their limits.

4. Conclusions

Kant embeds his third law of mechanics in a model of interaction he developed by constructive engagement with post-Leibnizian dynamics, rather than Newton's *Principia*. With his law, Kant repairs a paradigm whose broad outline Leibniz had endorsed explicitly. As a result, Kant's views of action and reaction become shaped by the post-Leibnizian milieu of its birth. This shows Leibniz's continued, though indirect, influence upon mid-18th century German natural philosophy. But it also complicates the project of reading the laws of Kant's a priori mechanics as fully equivalent to Newton's.

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⁹⁶ At *MAN* 4: 545, Kant explains that the active relation between the bodies is a matter of the line between them "according to the influence that the *motion* of one can have on the change of state of another" (my emphasis). This suggests he takes the action to be in the direction of the body's motion. Then he faces the task of explaining how bodies act when their motions are not in the same line.

⁹⁷ Newton to Roger Cotes, 31 March 1713, in *Edleston* (1969), pp. 15f; my italics.

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